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The name **thermodynamics** derives from the Greek **thermotis** (heat) and **dynamiki** (potential, power). Its historical roots are found in the quest to develop heat engines, devices that use heat to produce mechanical work. This quest, which was instrumental in powering the industrial revolution, gave birth to thermodynamics as a discipline that studies the relationship between heat, work, and energy.

Thermodynamics is the science that deals with the relationship of heat and mechanical energy and conversion of one into the other.

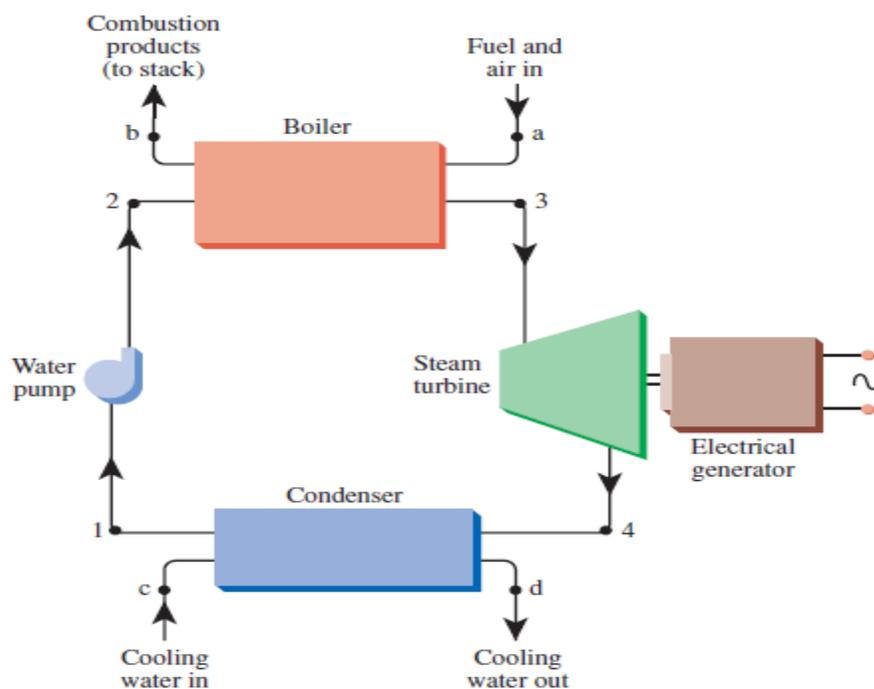


Fig. 1

The system is the part of the physical world that is the object of a thermodynamic calculation. It may be a fixed amount of material inside a tank, a gas compressor with the associated inlet and outlet streams, or an entire chemical plant. Once the system is defined, anything that lies outside the system boundaries belongs to the surroundings. In practice the matter contained within the boundaries of the system can be liquid, vapour, or gas and is known as the *working fluid*.

A system is a specifically identified fixed mass of material separated from its surroundings by a real or imaginary boundary.

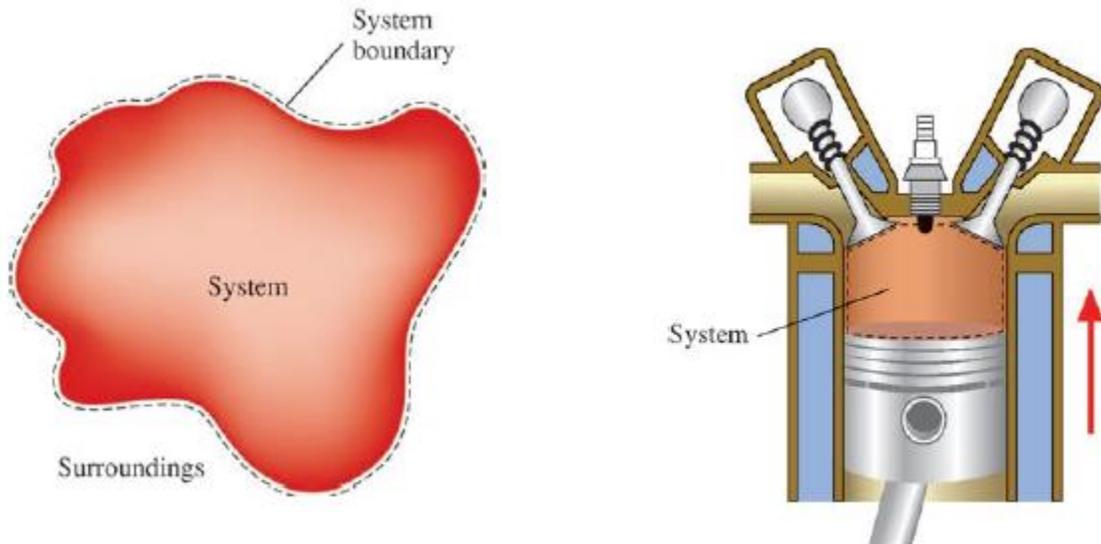


Fig. 2

A system can interact with its surroundings by exchanging mass, heat, and work. It is possible to construct the system in such way that some exchanges are allowed while others are not. If the system can exchange mass with the surroundings it called open, otherwise it is called closed.

A control volume is a region in space separated from its surroundings by a real

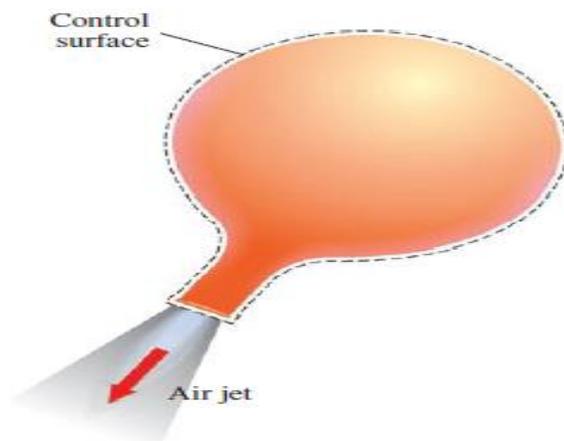


Fig. 3

or imaginary boundary, the control surface, across which mass may pass.

States:- A thermodynamic state of a system is defined by the values of all of the system thermodynamic properties.

Pressure is the ratio of the force acting normal to a surface, divided by the area of the surface. In thermodynamics, pressure generates the forces that give rise to mechanical work. The SI unit of pressure is the **Pascal, Pa**, and is defined as the pressure generated by 1 N (newton) acting on a 1 m² area:

$$Pa = \frac{F}{A} = \frac{N}{m^2} = \frac{J}{m^3}$$

$$P = h * \rho * g = \frac{N}{m^2} (Pa)$$

Where ρ is the liquid density (kg/m³)

The last equality in the far right is obtained by writing $J = N \cdot m$. The Pascal is an impractically small unit of pressure because 1 N is a small force and 1 m² is a large area. A commonly used multiple of Pa is the bar:

$$1 \text{ bar} = 10^5 \text{ Pa} = 100 \text{ kPa} = 0.1 \text{ MPa}.$$

$$1 \text{ atm} = \begin{cases} 1.013 \text{ bar} \\ 760 \text{ Torr} = 760 \text{ mmHg} \\ 14.696 \text{ psi} \end{cases}$$

Temperature is a fundamental property in thermodynamics. It is a measure of the kinetic energy of molecules and gives rise the sensation of “hot” and “cold.” It is measured using a thermometer, a device that obtains temperature indirectly by measuring some property that is a sensitive function of temperature, for example, the volume of mercury inside a capillary (mercury thermometer), the electric current between two different metallic wires (thermocouple), etc. In the SI system, the absolute temperature is a fundamental quantity (dimension) and its unit is the kelvin (K). In the American Engineering system, absolute temperature is measured in Rankine (R), whose relationship to the kelvin is,

$$1 \text{ K} = 1.8 \text{ R.}$$

$$T(^{\circ}\text{F}) = 1.8T(^{\circ}\text{C}) + 32$$

$$T(\text{K}) = T(^{\circ}\text{C}) + 273.15 \approx T(^{\circ}\text{C}) + 273$$

$$T(\text{R}) = T(^{\circ}\text{F}) + 459.67$$

Temperature is a parameter that characterizes the distribution of energies inside a system that is in equilibrium with its surroundings. With increasing temperature, the energy content of matter increases. Temperature, therefore, can be treated as a measure of the amount of energy stored inside matter.

Energy The SI unit of energy is the joule, defined as the work done by a force 1 Mover a distance of 1 m, also equal to the kinetic energy of a mass 1 kg with velocity 1 m/s:

$$J = N \cdot m = \frac{kg \cdot m^2}{s^2}$$

The kJ (1 kJ = 1000 J) is a commonly used multiple. These units are the Cal (calorie) and the Btu (British thermal unit) and are related to the joule through the following relationships: cures

$$\text{cal} = 4.18 \text{ J} = 4.18 \times 10^{-3} \text{ kJ}$$

$$\text{Btu} = 1.055 \times 10^3 \text{ J} = 1.055 \text{ kJ.}$$

Specific volume (v):- the specific volume of a system is the volume occupied by unit mass of the system the units are (m^3 / kg). the symbol V will be used for volume (m^3).

Work:- is defined as the product of a force and distance move in the direction of the force. The units of work are N m. if work done on unit mass of a fluid, then the work done per kg of fluid has units of N m/kg.

Processes:- A process occurs whenever a system changes from one state to another state.

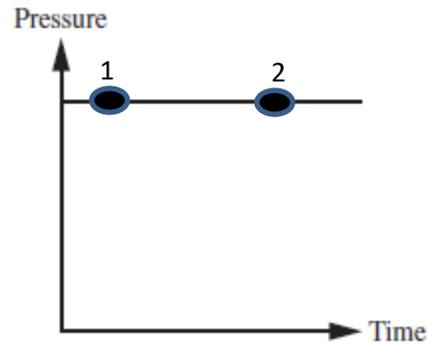
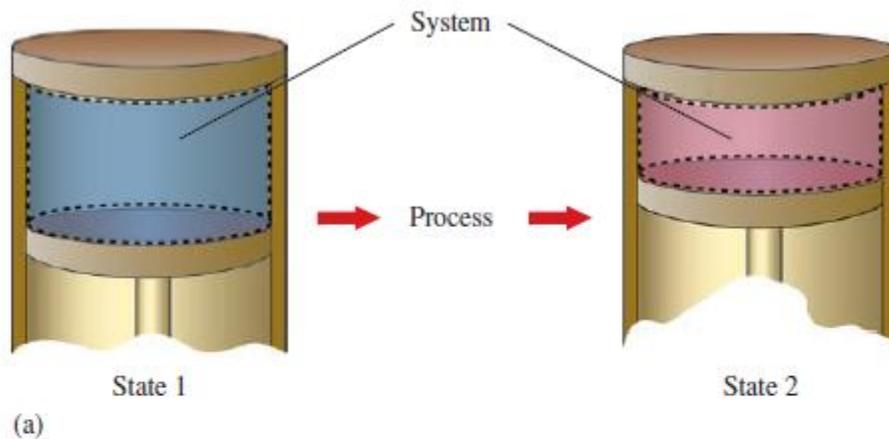


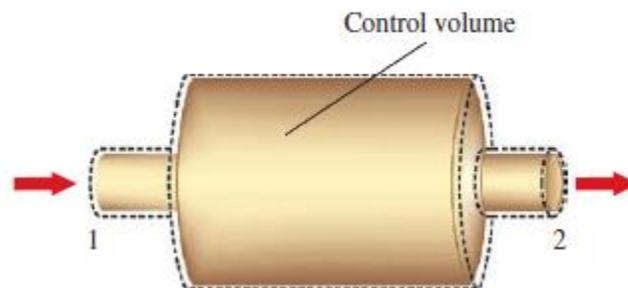
Fig. 5

Although formal definitions of process refer to systems, the term is also applied to control volumes, in particular, steady flows in which no properties change with time. We thus define a flow process as follows:

A flow process occurs whenever the state of the fluid entering a control volume is different from the state of the fluid exiting the control volume.



(a)



(b)

Fig. 5 A system undergoes a process that results in a change of the system state from state 1 to state 2. (b) In a steady-flow process, fluid enters the control volume at state 1 and exits the control volume at state 2.

When a system changes state in such a way that at any instant during the process the state point can be located on the diagram, then the process is said to be reversible (when a fluid undergoes a reversible process, both the fluid and its surroundings can always be restored to their original state). In practice, the fluid undergoing a process cannot be kept in equilibrium in its intermediate states and a continuous path cannot be traced on a diagram of properties. Such real processes are called irreversible processes. The criteria of reversibility are as follows:

- The process must be frictionless.
- The difference in pressure between the fluid and its surroundings during the process must be infinitely small.
- The difference in temperature between the fluid and its surroundings during the process must be infinitely small.

Cycles:- A thermodynamic cycle consists of a sequence of processes in which the working fluid returns to its original thermodynamic state.

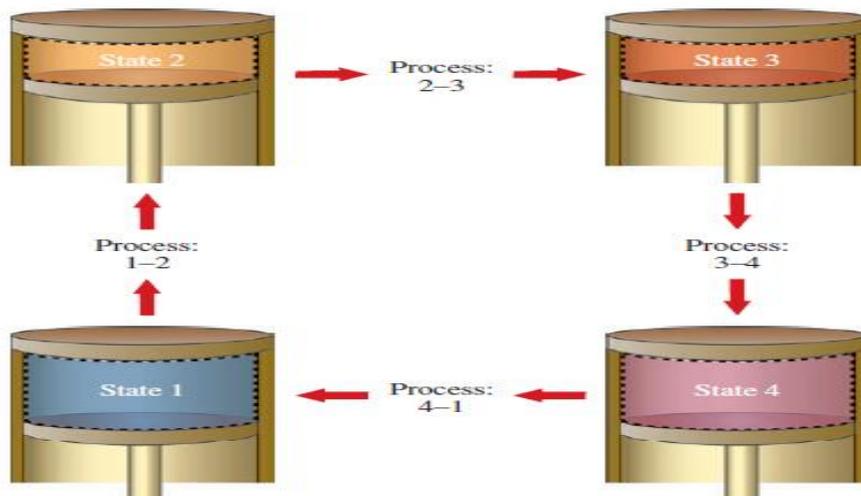
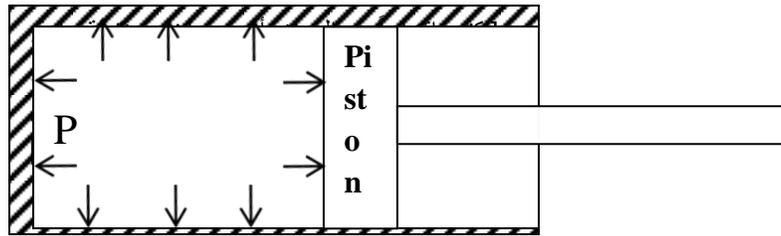


Fig. 6 A system undergoes a cycle when a series of processes returns the system to its original state. In this sketch, the cycle consists of the repetition of the state sequence 1–2–3–4–1.

Reversible work



Let the piston move under the action of the force exerted by the fluid a distance dl to the right. Then work done by the fluid on the piston is given by force times the distance moved.

$$\text{Work done by fluid} = (pA) * dl = pdV \text{ (J)}$$

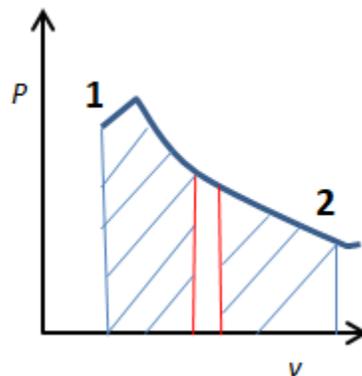
Where dV is a small increase in volume

Or considering unit mass

$$\text{Work done} = (pA) * dl = pdv \text{ (J/kg)}$$

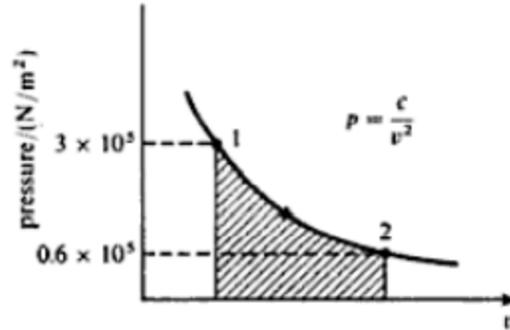
$$dv = A * dl$$

$$\text{Work done by unit mass of fluid} = \int_1^2 pdv \text{ (J/kg)}$$



Example 1

Unit mass of a fluid at a pressure of 3 bar, and with a specific volume of $0.18 \text{ m}^3/\text{kg}$, contained in a cylinder behind a piston expands reversibly to a pressure of 0.6 bar according to a law $p = c/v^2$, where c is a constant. Calculate the work done during the process.



$$W = -m \int_1^2 p \, dv = -m \text{ (shaded area)}$$

$$\text{i.e. } W = -cm \int_{v_1}^{v_2} \frac{dv}{v^2} = -cm \left[-\frac{1}{v} \right]_{v_1}^{v_2}$$

$$\text{also } c = pv^2 = 3 \times 0.18^2 = 0.0972 \text{ bar (m}^3/\text{kg)}^2$$

$$\text{and } v_2 = \sqrt{\frac{c}{p_2}} = \sqrt{\frac{0.0972}{0.6}} = 0.402 \text{ m}^3/\text{kg}$$

therefore

$$\begin{aligned} W &= -0.0972 \times 10^5 \left(\frac{1}{0.18} - \frac{1}{0.402} \right) \text{ N m/kg} \\ &= -29\,840 \text{ N m/kg} \end{aligned}$$

$$\text{i.e. } \text{Work done by the fluid} = +29\,840 \text{ N m/kg}$$

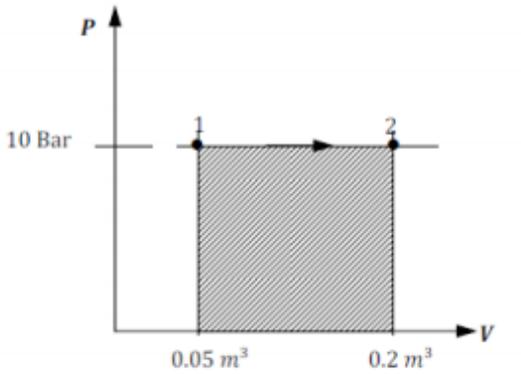
Problems

A certain fluid at 10 bar is contained in a cylinder behind a piston, the initial volume being 0.05 m^3 . Calculate the work done by the fluid when it expands reversibly:

- (i) at constant pressure to a final volume of 0.2 m^3 ;
- (ii) according to a linear law to a final volume of 0.2 m^3 and a final pressure of 2 bar;
- (iii) according to a law $pV = \text{constant}$ to a final volume of 0.1 m^3 ;
- (iv) according to a law $pv^3 = \text{constant}$ to a final volume of 0.06 m^3 ;
- (v) according to a law, $p = (A/V^2) - (B/V)$, to a final volume of 0.1 m^3 and a final pressure of 1 bar, where A and B are constants.

Solution

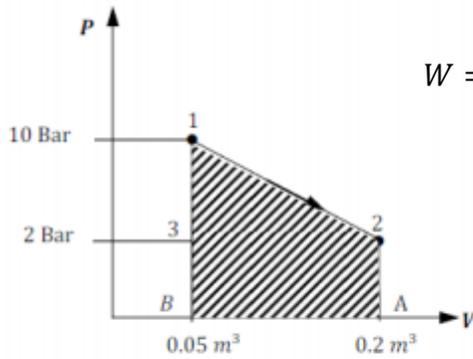
i.



For reversible non-flow process we have

$$W = \text{shaded area} = \int_{V_1}^{V_2} p dV = p(V_2 - V_1) = 1000 \text{ kPa} \cdot (0.2 - 0.05) \text{ m}^3 = 150 \text{ kJ}.$$

ii.



$$W = \text{shaded area} = \left(\frac{p_1 + p_2}{2}\right)(V_2 - V_1) + p_2(V_2 - V_1)$$

$$W = \text{shaded area} = \left(\frac{p_1 + p_2}{2}\right) |V_2 - V_1| = \left(\frac{10 + 2}{2}\right) 10^5 |0.2 - 0.05| = 90 \text{ kJ}.$$

CH2

2.1 First Law of Thermodynamics (Conservation of energy).

Energy is a fundamental concept of thermodynamics and one of the most significant aspects of engineering analysis. Energy can be stored within systems in various macroscopic forms: kinetic energy, gravitational potential energy, and internal energy (neither created nor destroyed). Energy can also be transformed from one form to another and transferred between systems. For closed systems, energy can be transferred by work and heat transfer. The total amount of energy is conserved in all transformations and transfers.

When a system undergoes a thermodynamic cycle then the net heat supplied to the system from its surroundings is equal to the net work done by the system on its surroundings.

$$\sum dQ = \sum dW$$

Where \sum represents the sum for a complete cycle.

2.2 the non-flow energy equation.

In the previous section

$$\sum dQ = \sum dW$$

The final intrinsic energy of the system is equal to its initial value. Consider now a process which the intrinsic energy of the system is finally greater than the initial intrinsic energy.

$$\text{Gain in intrinsic energy} = \sum dQ - \sum dW$$

Where a fluid is not in motion

$$\text{Gain in intrinsic energy} = \text{internal energy } (u \text{ kJ/kg})$$

$$U = mu \text{ kJ}$$

$$U_2 - U_1 = \sum_1^2 dQ - \sum_1^2 dW$$

$U_2 - U_1 = Q - W$ for a non-flow process

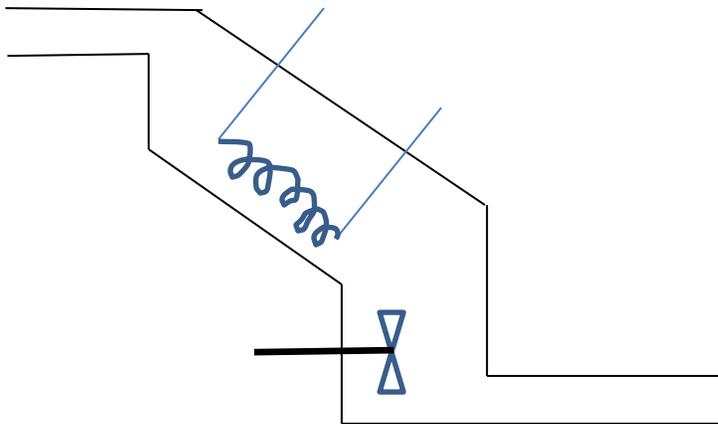
$$Q = (U_2 - U_1) + W \text{ kJ}$$

$$Q = (u_2 - u_1) + W \text{ kJ/kg}$$

$$dQ = du + dW$$

2.3 The flow energy equation.

The flow equation in the last section, the internal energy of a fluid was said to be the intrinsic energy of the fluid due to its thermodynamic properties.



Specific internal energy, u —is the energy stored in the substance due to molecular motion as well as intermolecular forces. The SI unit is kJ/kg.

Kinetic energy, KE —is the energy a system of mass m possesses due to the macro fluid motion at velocity C .

$$KE = mC^2/2$$

Potential energy, PE —is the energy due to the gravitational field g that a mass m possess in relation to a height z from a reference plane.

$$PE = mgz$$

Shaft work, W —is the mechanical work produced or absorbed by the rotating shaft of the thermal machine.

Heat, Q —is the form of energy transferred to or from the machine due to a difference of temperatures between the machine and the surroundings, the higher temperature to the lower one.

Energy required to push element across boundary = $(p_1 A_1) * l$

$$= p_1 (\text{volume of fluid element})$$

Energy required for 1 kg of fluid = $p_1 v_1$

Where v_1 is the specific volume of the fluid at section 1

Energy required at exit to push 1 kg of fluid = $p_2 v_2$

Steady flow energy equation

$$u_1 + \frac{C_1^2}{2} + Z_1 g + p_1 v_1 + Q = u_2 + \frac{C_2^2}{2} + Z_2 g + p_2 v_2 + W$$

Specific enthalpy, h —is the sum of the specific internal energy and the product

of pressure P versus specific volume, v . The SI unit is kJ/kg.

$$h = u + Pv$$

the enthalpy of mass, m , $H = mh$

$$h_1 + \frac{C_1^2}{2} + Z_1g + Q = h_2 + \frac{C_2^2}{2} + Z_2g + W$$

In nearly all problems in applied, changes in height are negligible and the potential energy terms can be omitted from equation.

$$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W$$

This equation called the steady-flow energy equation

Consider any section of cross-sectional area is A , the fluid velocity is C , then the rate of volume flow past section is CA . Also, since mass flow is volume flow divided by specific volume,

$$\text{mass flow rate, } \dot{m} = \frac{CA}{v} \left(\frac{kg}{s} \right)$$

Where v is the specific volume of fluid at the section.

$$\text{mass flow rate, } \dot{m} = \frac{C_1A_1}{v_1} = \frac{C_2A_2}{v_2}$$

CH 3.

The working fluid.

Behavior of the working substance is very essential factor in understanding thermodynamics. In this chapter, focus is given to pure substances such as gases and steam properties and how they are interrelated are important in the design and operation of thermal systems.

3.1 Liquid, vapour, and gas.

Consider a p-v diagram for any substance. The solid phase is not important in engineering thermodynamics. When a liquid is heated at any one constant pressure is one fixed temperature at which bubbles of vapour form in the liquid, this phenomenon is known as boiling. The higher the pressure of the liquid then the higher the temperature at which boiling occurs. It is also found that the volume occupied by 1 kg of a boiling liquid at a higher pressure is slightly larger than the volume occupied by 1 kg of the same liquid when it is boiling at a low pressure.

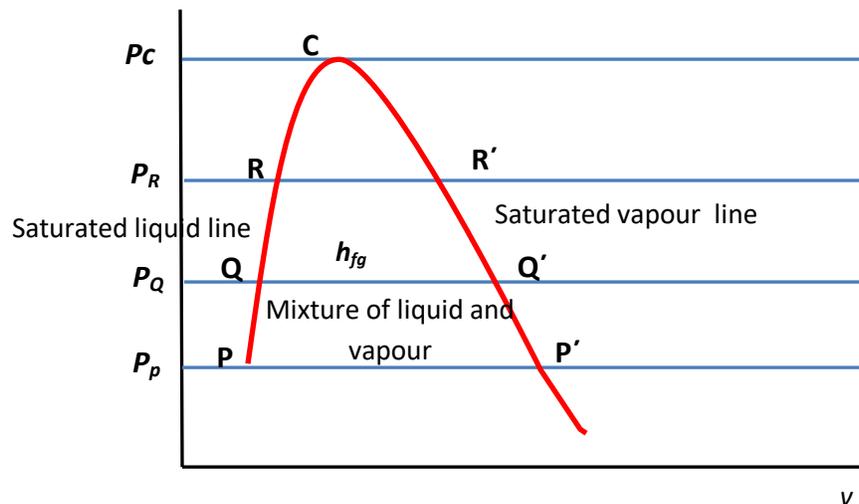


Fig. 3.1

The point P, Q and R represent the boiling points of a liquid at pressure P, Q, and R respectively. When a liquid at boiling point is heated further at constant pressure the additional heat supplied changes the phase of substance from liquid to vapour,

during this change of phase the pressure and temperature remain constant. The heat supplied is called the latent heat of vaporization. It is found that the higher the pressure then the smaller is the amount of latent heat required. There is a definite value of specific volume of the vapour at any one pressure, at the point at which vaporization is complete, hence a series of points such as P', Q' and R' can be plotted and joined to form a line. The pressure at which the turning point occurs is called the critical pressure and the turning point itself is called the critical point. It can be seen that at the critical point the latent heat zero.

A saturated vapour is usually called dry saturated to emphasize the fact that no liquid is present in the vapour in this state. Lines of constant temperature, called isothermals, can be plotted on a p-v diagram.

When a dry saturated vapour is heated at constant pressure its temperature raises and it becomes superheated. The difference between the actual temperature of the superheated vapour and the saturation temperature at the pressure of the vapour is called the degree of superheat.

Dryness fraction, x = the mass of dry vapour in kg of the mixture

Wetness fraction, y = the mass of liquid in 1 kg of the mixture ($y = 1 - x$)

$$v = \frac{\text{volume of liquid} + \text{volume of dry vapour}}{\text{total mass of wet vapour}}$$

$$v = v_f(1 - x) + v_g x$$

The volume of the liquid is usually negligibly small compared to the volume of dry saturated vapour.

$$v = v_g x$$

The enthalpy of a wet vapour

$$h = h_f(1 - x) + h_g x$$

$$\therefore h = h_f + x(h_g - h_f)$$

$$h = h_f + x h_{fg}$$

The internal energy

$$u = u_f + x(u_g - u_f)$$

3.2 The perfect gas

The characteristic equation of state

At temperature that are considerably in excess of the critical temperature of a fluid, and also at very low pressure, the vapour of the fluid tends to obey the equation.

$$\frac{pv}{T} = \text{constant} = R$$

An imaginary ideal gas which obeys the law is called a perfect gas, and the equation, $pv/T=R$, is called the characteristic equation of state of a perfect gas. The constant, R , is called the gas constant.

$$\text{the unit of } R = N \text{ m/kg K}$$

The characteristic equation is usually written

$$pv = TR$$

For m kg, occupying $V \text{ m}^3$,

$$pV = mTR$$

$$m = nM$$

Where n is the number of moles, and M is the molecular weight of the gas (kg/mol).

$$\therefore pV = nMTR \text{ or } MR = \frac{pV}{nT}$$

Avogadro's hypotheses state that the volume of 1 mole of any gas is the same as the volume of 1 mole of any other gas, when the gases are at the same temperature and pressure.

$$\therefore \frac{V}{n} \text{ is the same for all gases at the same value of } p \text{ and } T$$

$\therefore pV/nT$ is a constant for all gases, this constant called universal gas constant (R_0)

$$\therefore MR = R_0 = \frac{pV}{nT} \text{ or } pV = nR_0T$$

$$R = \frac{R_0}{M}$$

$$\therefore R_0 = 8314.3 \frac{Nm}{\text{mole } K} \text{ for 1 mole at 1 bar and } 0^\circ\text{C}$$

Specific heat

The specific heat of a solid or liquid is usually defined as the heat required to raise unit mass through one degree temperature rise.

$$\therefore \text{for small quantities } dQ = mcdT$$

Where m is the mass, dT is the increase in temperature and c is the specific heat. Only two specific heat for gases are defined; the specific heat at constant volume C_v , and the specific heat at constant pressure C_p . We have,

$$\therefore dQ = mc_p dT \text{ for a reversible non flow process at constant pressure}$$

$$\therefore dQ = mc_v dT \text{ for a reversible non flow process at constant volume}$$

For a perfect gas the values of c_p and c_v are constant for any one gas at all pressure and temperature.

$$\begin{aligned} \therefore \text{Heat flow in a reversible constant pressure process} &= \int_{T_1}^{T_2} mc_p dT \\ &= mc_p (T_2 - T_1) \end{aligned}$$

$$\begin{aligned} \therefore \text{Heat flow in a reversible constant volume process} &= \int_{T_1}^{T_2} mc_v dT \\ &= mc_v (T_2 - T_1) \end{aligned}$$

Joule's law

Joule's law states that the internal energy of a perfect gas is a function of the absolute temperature only, $u = f(T)$. To evaluate this function for 1 kg at $v = C$.

$$dQ = du + dW$$

$$\because v = C \therefore dW = 0$$

$$\therefore dQ = du$$

At constant volume for perfect gas.

$$\therefore dQ = c_v dT$$

Therefore, $dQ = du = c_v dT$, and integrating,

$$\therefore u = c_v T + K, \text{ where } K \text{ is a constant}$$

For a perfect gas it can be assumed that $u = 0$ when $T = 0$, hence the constant $K = 0$.

$$\therefore u = c_v T, \text{ or for mass, } m, U = mc_v T$$

$$\therefore \text{Gain in internal energy, } U_2 - U_1 = mc_v(T_2 - T_1)$$

Relationship between the specific heats

Let a perfect gas be heated a constant pressure from T_1 to T_2 for non- flow process,

$$Q = (U_2 - U_1) + W$$

$$\therefore Q = mc_v(T_2 - T_1) + W$$

$$\because p = C. \therefore W = p(V_2 - V_1), \because pV = mRT$$

$$\therefore W = mR(T_2 - T_1)$$

$$\therefore Q = mc_v(T_2 - T_1) + mR(T_2 - T_1) = m(c_v + R)(T_2 - T_1)$$

But for a constant pressure process, $Q = mc_p(T_2 - T_1)$.

$$\therefore mc_p(T_2 - T_1) = m(c_v + R)(T_2 - T_1)$$

$$\therefore c_p = c_v + R$$

$$\therefore c_p - c_v = R$$

Enthalpy of perfect gas

Enthalpy,

$$\therefore h = u + pv$$

$$\therefore u = c_v T, pv = RT$$

$$\therefore h = c_v T + RT = (c_v + R)T, \therefore c_p = (c_v + R)$$

$$\therefore h = c_p T$$

For mass, m of a perfect gas, $H = mc_p T$

Ratio of specific heats

The ratio of the specific heat at constant pressure to the specific heat at constant volume is given the symbol γ (*gamma*)

$$\gamma = \frac{c_p}{c_v}$$

$$\therefore c_p - c_v = R, \text{ dividing through by } c_v$$

$$\therefore \frac{c_p}{c_v} - 1 = \frac{R}{c_v}$$

Therefore

$$\therefore \gamma - 1 = \frac{R}{c_v}, \text{ and } c_v = \frac{R}{\gamma - 1}$$

$$\therefore c_p = \frac{\gamma R}{(\gamma - 1)}$$

Example

A vessel of volume 0.2 m^3 contains nitrogen at 1.013 bar and 15°C . If 0.2 kg of nitrogen is now pumped into the vessel, calculate the new pressure when the vessel has returned to its initial temperature. The molar mass of nitrogen is 28 kg/kmol , and it may be assumed to be a perfect gas.

Solution

$$\text{Specific gas constant, } R = \frac{\bar{R}}{\bar{m}} = \frac{8314.5}{28} = 296.95 \text{ N m/kg K}$$

$$p_1 V_1 = m_1 R T_1$$

therefore

$$m_1 = \frac{p_1 V_1}{R T_1} = \frac{1.013 \times 10^5 \times 0.2}{296.95 \times 288} = 0.237 \text{ kg}$$

where $T_1 = 15 + 273 = 288 \text{ K}$

The mass of nitrogen added is 0.2 kg , hence $m_2 = 0.2 + 0.237 = 0.437 \text{ kg}$. Then from equation (2.6), for the final conditions

$$p_2 V_2 = m_2 R T_2$$

but $V_2 = V_1$ and $T_2 = T_1$, therefore

$$p_2 = \frac{m_2 R T_2}{V_2} = \frac{0.437 \times 296.95 \times 288}{10^5 \times 0.2}$$

i.e. $p_2 = 1.87 \text{ bar}$

Example

A certain perfect gas of mass 0.01 kg occupies a volume of 0.003 m^3 at a pressure of 7 bar and a temperature of 131°C . The gas is allowed to expand until the pressure is 1 bar and the final volume is 0.02 m^3 . Calculate:

- (i) the molar mass of the gas;
- (ii) the final temperature.

Solution

$$p_1 V_1 = m R T_1$$

therefore

$$R = \frac{p_1 V_1}{m T_1} = \frac{7 \times 10^5 \times 0.003}{0.01 \times 404} = 520 \text{ N m/kg K}$$

where $T_1 = 131 + 273 = 404 \text{ K}$.

$$R = \frac{\bar{R}}{\bar{m}}$$

therefore

$$\bar{m} = \frac{\bar{R}}{R} = \frac{8314.5}{520} = 16 \text{ kg/kmol}$$

i.e. Molar mass = 16 kg/kmol

$$p_2 V_2 = m R T_2$$

therefore

$$T_2 = \frac{p_2 V_2}{m R} = \frac{1 \times 10^5 \times 0.02}{0.01 \times 520} = 384.5 \text{ K}$$

i.e. Final temperature = $384.5 - 273 = 111.5^\circ\text{C}$

Example

A perfect gas has a molar mass of 26 kg/kmol and a value of $\gamma = 1.26$. Calculate the heat rejected:

- (i) when unit mass of the gas is contained in a rigid vessel at 3 bar and 315 °C, and is then cooled until the pressure falls to 1.5 bar;
- (ii) when unit mass flow rate of the gas enters a pipeline at 280 °C, and flows steadily to the end of the pipe where the temperature is 20 °C. Neglect changes in velocity of the gas in the pipeline.

Solution

$$R = \frac{\bar{R}}{\bar{m}} = \frac{8314.5}{26} = 319.8 \text{ N m/kg K}$$

From equation (2.21)

$$c_v = \frac{R}{(\gamma - 1)} = \frac{319.8}{10^3(1.26 - 1)} = 1.229 \text{ kJ/kg K}$$

$$\frac{c_p}{c_v} = \gamma$$

therefore

$$c_p = \gamma c_v = 1.26 \times 1.229 = 1.548 \text{ kJ/kg K}$$

(i) The volume remains constant for the mass of gas present, and hence the specific volume remains constant.

$$p_1 v_1 = RT_1 \quad \text{and} \quad p_2 v_2 = RT_2$$

Therefore since $v_1 = v_2$ we have

$$T_2 = T_1 \frac{p_2}{p_1} = 588 \times \frac{1.5}{3} = 294 \text{ K}$$

where $T_1 = 315 + 273 = 588 \text{ K}$.

Then from equation (2.13)

$$\begin{aligned} \text{Heat supplied per kg of gas} &= c_v(T_2 - T_1) = 1.229(294 - 588) \\ &= -1.229 \times 294 = -361 \text{ kJ/kg} \end{aligned}$$

i.e. Heat rejected per kilogram of gas = +361 kJ/kg

(ii) From the steady-flow energy equation, (1.10),

$$\dot{m} \left(h_1 + \frac{C_1^2}{2} \right) + \dot{Q} + \dot{W} = \dot{m} \left(h_2 + \frac{C_2^2}{2} \right)$$

In this case we are told that changes in velocity are negligible; also there is no work done.

Therefore we have

$$\dot{m}h_1 + \dot{Q} = \dot{m}h_2$$

For a perfect gas, from equation (2.18)

$$h = c_p T$$

therefore

$$\dot{Q} = \dot{m}c_p(T_2 - T_1) = 1 \times 1.548(20 - 280) = -403 \text{ kW}$$

i.e. Heat rejected per kilogram per second = +403 kW

Note that it is not necessary to convert $t_1 = 280 \text{ °C}$ and $t_2 = 20 \text{ °C}$ into degrees Kelvin, since the temperature difference ($t_1 - t_2$) is numerically the same as the temperature difference ($T_1 - T_2$).

Ch.- 4

Reversible and irreversible process

4-1 Reversible non-flow process

Polytropic Process

$$pv^n = \text{constant}$$

Where n is the polytropic coefficient. It can assume any value. Some particular values of n represent a special thermodynamic process, such as:

- Isobaric process ($p = \text{constant}$): $n = 0$;
- Isothermal process ($T = \text{constant}$): $n = 1$;
- Isentropic or adiabatic reversible process ($Q=0, s = \text{constant}$): $n = \gamma$;
- Isochoric process ($v = \text{constant}$): $n = \infty$?

A- Isochoric process (constant volume process, $v = \text{constant}$)

In a constant volume process the working substance is contained in a rigid vessel, hence the boundaries of a system are immovable and no work can be done on or by the system ($w=0$).

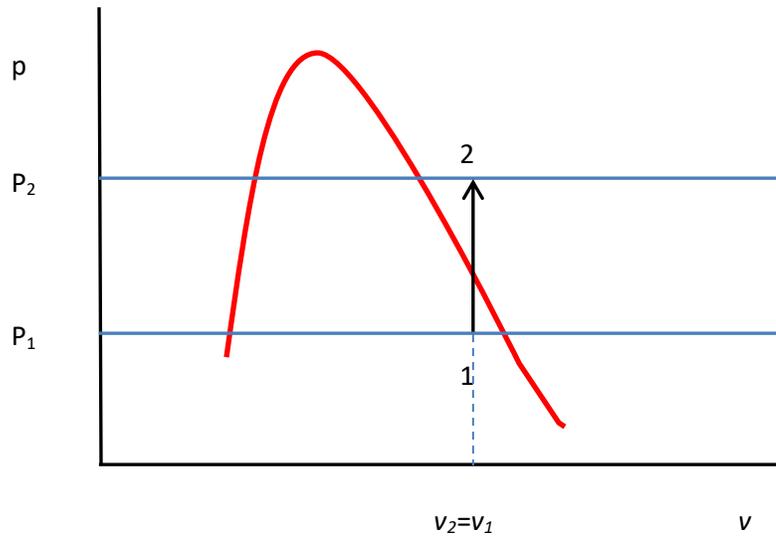
The non-flow equation,

$$Q = (u_2 - u_1) + w$$

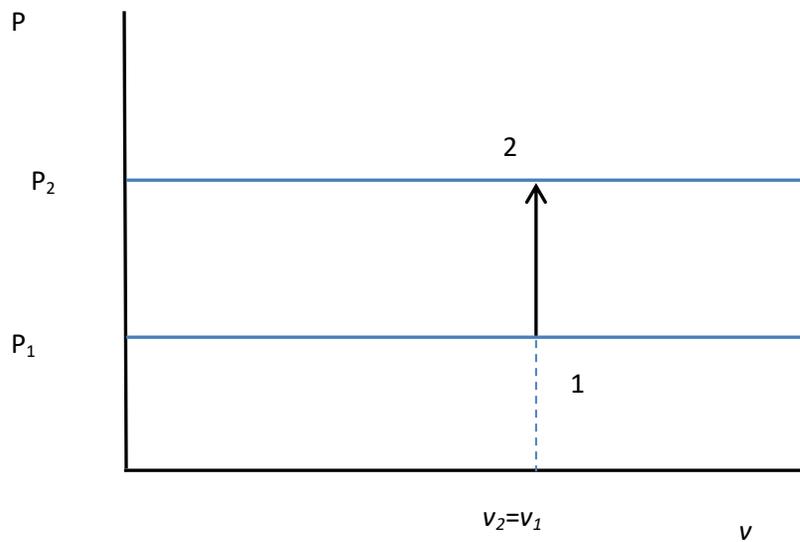
$$\therefore w = 0$$

$$\therefore Q = (u_2 - u_1) \text{ or for mass, } m Q = U_2 - U_1$$

A constant volume process for a vapour is shown on a p - v diagram.



For vapour



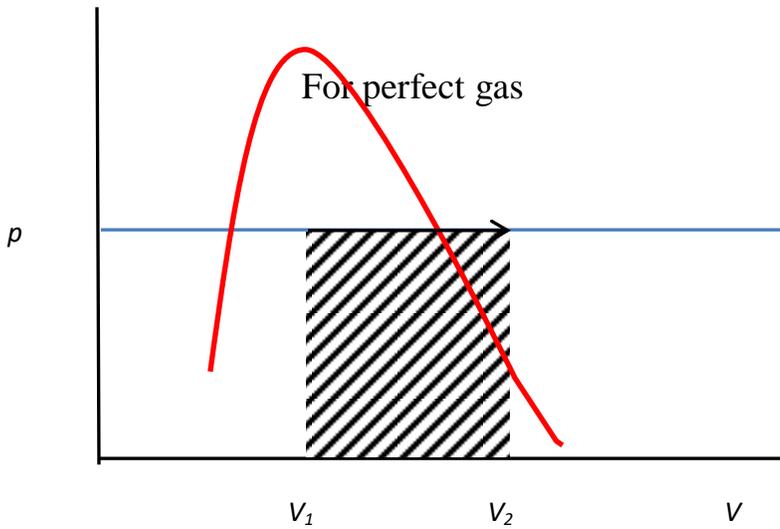
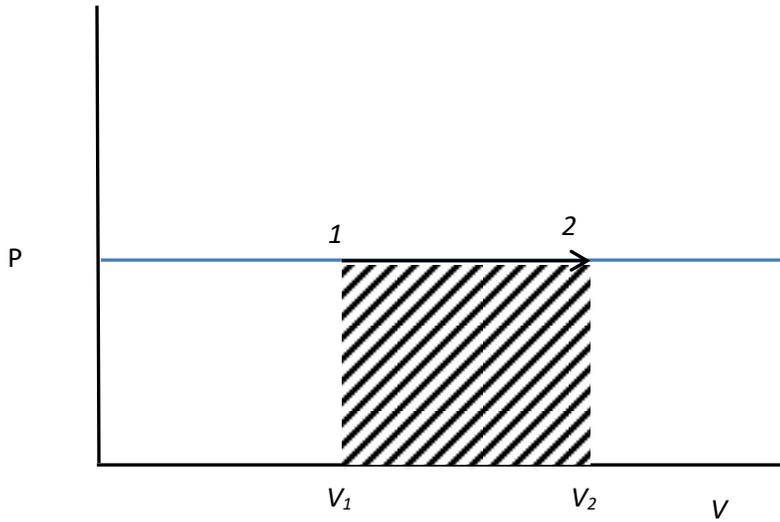
For perfect gas

For perfect gas we have from

equation

$$Q = mc_v(T_2 - T_1)$$

B- Constant pressure process (Isobaric process ($p = \text{constant}$): $n = 0$).



For vapour

$$W = \int_{v_1}^{v_2} p \, dv \text{ for any reversible process}$$

$$\therefore W = p \int_{v_1}^{v_2} dv = p(v_2 - v_1) \text{ when } p = \text{constant}$$

From the non-flow energy equation,

$$Q = (u_2 - u_1) + w$$

Hence for a reversible constant pressure process,

$$Q = (u_2 - u_1) + p(v_2 - v_1) = (u_2 + pv_2) - (u_1 + pv_1)$$

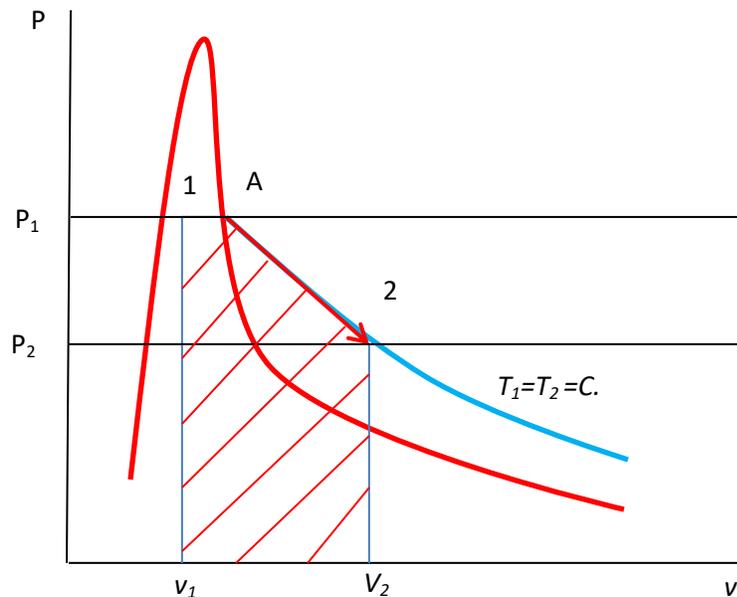
$$\therefore h = (pv + u)$$

$$\therefore Q = (h_2 - h_1) \text{ for mass, } m, Q = H_2 - H_1$$

For perfect gas

$$Q = m c_p (T_2 - T_1)$$

C- Constant temperature (Isothermal process ($T = \text{constant}$): $n = 1$)



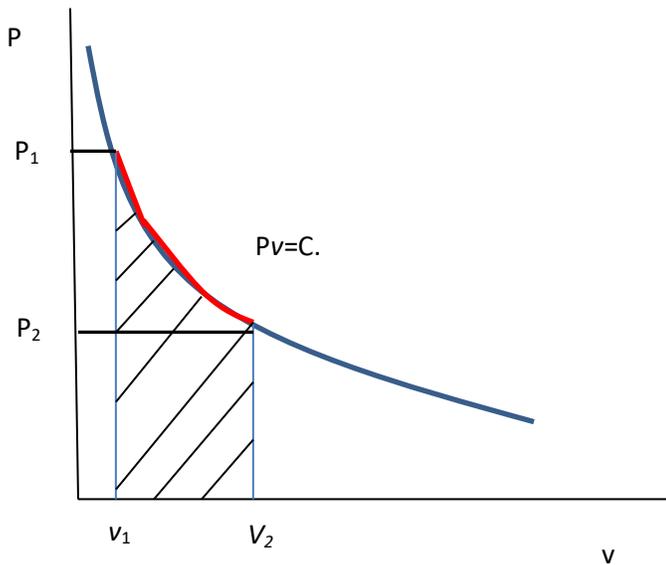
$$pv^1 = \text{constant}, p = c/v$$

$$Q = (u_2 - u_1) + w = (u_2 - u_1) + c \ln \frac{v_2}{v_1}$$

$$W = \int_{v_1}^{v_2} p dv = \int_{v_1}^{v_2} c \frac{dv}{v} = c [\ln v]_{v_1}^{v_2} = c \ln \frac{v_2}{v_1}$$

$C = \text{constant}$, $p_1 v_1 = p_2 v_2 = c$.

An isothermal process for a perfect gas



$$pv = RT$$

$$\therefore T = \text{Constant}$$

$$pv = RT = \text{constant}$$

Therefore, for an isothermal process for a perfect gas

$$pv = \text{constant} = RT$$

$$W = \int_1^2 p dv$$

$$W = \int_{v_1}^{v_2} c \frac{dv}{v} = c [\ln v]_{v_1}^{v_2} = c \ln \frac{v_2}{v_1}$$

C.=constant, $p_1 v_1 = p_2 v_2 = RT = c$.

$$W = p_1 v_1 \text{ or } p_2 v_2 \ln \frac{v_2}{v_1} \text{ per unit mass of gas}$$

$$W = m P_1 V_1 \text{ or } P_2 V_2 \ln \frac{v_2}{v_1} \text{ for mass, } m \text{ of the gas}$$

Since $p_1 v_1 = p_2 v_2$

$$\frac{v_2}{v_1} = \frac{p_1}{p_2}$$

$$\therefore W = p_1 v_1 \text{ or } p_2 v_2 \ln \frac{p_1}{p_2} \text{ per unit mass, } m, \text{ of gas}$$

$$W = m p_1 V_1 \text{ or } p_2 V_2 \ln \frac{p_1}{p_2} \text{ for mass, } m \text{ of gas}$$

$$\therefore p_1 v_1 = RT$$

$$W = RT \ln \frac{p_1}{p_2} \text{ per unit mass, } m, \text{ of gas}$$

$$W = p_1 v_1 \text{ or } p_2 v_2 \ln \frac{v_2}{v_1} \text{ per unit mass of gas}$$

$$W = mRT \ln \frac{p_1}{p_2} \text{ for mass, } m, \text{ of gas}$$

For a perfect gas from Joules law,

$$U_2 - U_1 = m c_v (T_2 - T_1)$$

$$\therefore T_2 = T_1$$

$$U_2 - U_1 = 0$$

From the non-flow energy equation,

$$Q = c_v(T_2 - T_1) \text{ or } (u_2 - u_1) + W$$

$$Q = W$$

4.2 reversible non-flow equation Isentropic or adiabatic reversible process ($Q=0$, $s = \text{constant}$): $n = c$:-

An adiabatic process is one in which no heat is transferred to or from the fluid during process.

From the non-flow equation.

$$Q = (u_2 - u_1) + w$$

And for adiabatic process

$$Q = 0$$

Therefore, we have

$$W = (u_1 - u_2) \text{ for adiabatic process}$$

In an adiabatic expansion, the work done by the fluid is at the expense of a reduction in the internal energy of the fluid. Similarly, in an adiabatic compression process all the work done on the fluid goes to increasing the internal energy of the fluid.

For perfect gas

$$dQ = du + dw$$

Also for a reversible process $dW = pdv$, hence,

$$dQ = du + pdv$$

Since $Q = 0$ for an adiabatic process.

Now for perfect gas $pv = RT$ or $p = \frac{RT}{v}$

Hence substituting

$$du + \frac{RT dv}{v} = 0$$

Also for perfect gas

$$u = c_v T \text{ or } du = c_v dT$$

$$\therefore c_v dT + \frac{RT dv}{v} = 0$$

Dividing through by T

$$\text{i. e. } c_v \frac{dT}{T} + \frac{R dv}{v} = 0$$

Integrating

$$c_v \ln T + R \ln v = \text{constant}$$

then $T = \frac{pv}{R}$, therefore substituting

$$c_v \ln \frac{pv}{R} + R \ln v = \text{constant}$$

Dividing by c_v

$$\ln \frac{pv}{R} + \frac{R}{c_v} \ln v = \text{constant}$$

$$\text{also } c_v = \frac{R}{(\gamma - 1)} \text{ or } \frac{R}{c_v} = (\gamma - 1)$$

Hence substituting

$$\ln \frac{pv}{R} + (\gamma - 1) \ln v = \text{constant}$$

$$\ln \frac{pv}{R} + \ln v^{(\gamma-1)} = \text{constant}$$

$$\therefore \ln \frac{pv v^{(\gamma-1)}}{R} = \text{constant}$$

$$i.e \ln \frac{pv^\gamma}{R} = \text{constant}$$

$$i.e \frac{pv^\gamma}{R} = e^{\text{constant}} = \text{constant or } pv^\gamma = \text{constant}$$

$$\therefore p = \frac{RT}{v}$$

Substituting in, $pv^\gamma = \text{constant}$

$$\frac{RTv^\gamma}{v} = \text{constant}$$

$$i.e. Tv^{(\gamma-1)} = \text{constant}$$

Also, $v = RT/p$

$$p \left(\frac{RT}{p} \right)^\gamma = \text{constant}$$

$$\therefore \frac{T^\gamma}{p^{(\gamma-1)}} = \text{constant}$$

$$\therefore \frac{T}{p^{(\gamma-1)/\gamma}} = \text{constant}$$

Therefore, for a reversible adiabatic process for a perfect gas between state 1 to 2 we can write:

$$\therefore p_1 v_1^\gamma = p_2 v_2^\gamma \text{ or } \frac{p_1}{p_2} = \left(\frac{v_2}{v_1} \right)^\gamma$$

$$\text{also } T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1} \text{ or } \frac{T_1}{T_2} = \left(\frac{v_2}{v_1} \right)^{\gamma-1}$$

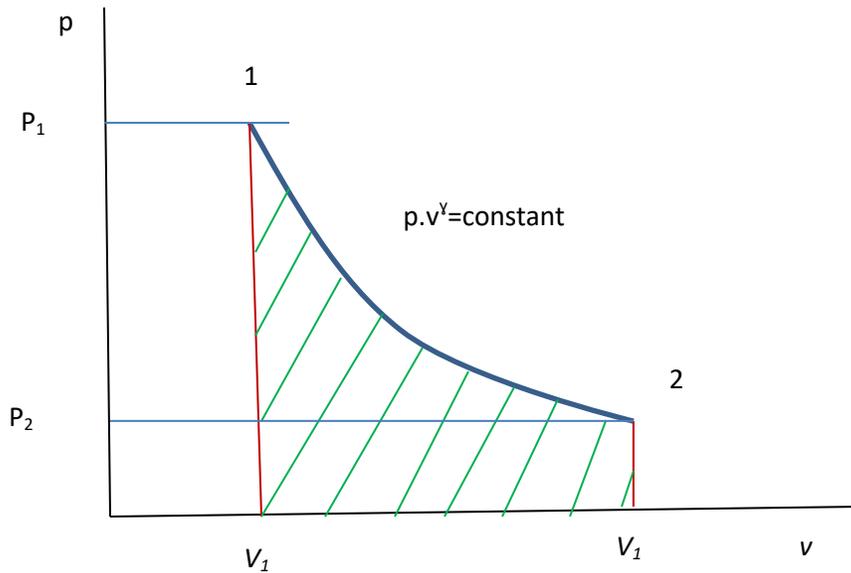
$$\therefore \frac{T_1}{p_1^{(\gamma-1)/\gamma}} = \frac{T_2}{p_2^{(\gamma-1)/\gamma}} \text{ or } \frac{T_1}{T_2} = \left(\frac{p_1}{p_2} \right)^{(\gamma-1)/\gamma}$$

The work done in an adiabatic process for gas.

$$\therefore W = (u_1 - u_2) \quad \because u_1 - u_2 = c_v(T_1 - T_2)$$

$$\therefore W = c_v(T_1 - T_2)$$

$$\therefore \text{also } c_v = \frac{R}{\gamma - 1}$$



For perfect gas

$$\therefore W = \frac{R}{\gamma - 1}(T_1 - T_2)$$

$$\because pv = RT \quad \therefore W = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$$

The work done on or by perfect gas for adiabatic process is shown in above p-v diagram.

$$W = \int_{v_1}^{v_2} p dv$$

$$\therefore W = \int_{v_1}^{v_2} \frac{c}{v^\gamma} dv = c \left[\frac{v^{-\gamma+1}}{-\gamma+1} \right]_{v_1}^{v_2} = c \left(\frac{v_1^{-\gamma+1} - v_2^{-\gamma+1}}{\gamma-1} \right)$$

$$\therefore p_1 v_1^\gamma = p_2 v_2^\gamma = \text{constant}$$

$$\therefore W = \left(\frac{p_1 v_1^\gamma v_1^{-\gamma+1} - p_2 v_2^\gamma v_2^{-\gamma+1}}{\gamma-1} \right)$$

$$\therefore W = \frac{p_1 v_1 - p_2 v_2}{\gamma-1}$$

4.3 polytropic process (pv^n=constant):-

$$W = \int p dv$$

$$\therefore p = c/v^n$$

$$\therefore W = \int_{v_1}^{v_2} \frac{c}{v^n} dv = c \left[\frac{v^{-n+1}}{-n+1} \right]_{v_1}^{v_2} = c \left(\frac{v_1^{-n+1} - v_2^{-n+1}}{n-1} \right)$$

$$\therefore p_1 v_1^n = p_2 v_2^n = \text{constant}$$

$$\therefore W = \left(\frac{p_1 v_1^n v_1^{-n+1} - p_2 v_2^n v_2^{-n+1}}{n-1} \right)$$

$$\therefore W = \frac{p_1 v_1 - p_2 v_2}{n-1} \text{ for any working fluid}$$

$$\therefore p_1 v_1^n = p_2 v_2^n \text{ or } \frac{p_1}{p_2} = \left(\frac{v_2}{v_1} \right)^n$$

For perfect gas

$$pv = RT \text{ or } p = \frac{RT}{v}, \text{ hence substituting in } pv^n = c, \text{ we have}$$

$$\therefore \frac{RT}{v} v^n = C \text{ or } T v^{n-1} = C.$$

also $v = RT/p$

$$p \left(\frac{RT}{p} \right)^n = \text{constant}$$

$$\therefore \frac{T^n}{p^{(n-1)}} = \text{constant}$$

$$\therefore \frac{T}{p^{(n-1)/n}} = \text{constant}$$

Therefore, for a reversible apolytropic process for a perfect gas between state 1 to 2 we can write:

$$\therefore p_1 v_1^n = p_2 v_2^n \text{ or } \frac{p_1}{p_2} = \left(\frac{v_2}{v_1} \right)^n$$

$$\text{also } T_1 v_1^{n-1} = T_2 v_2^{n-1} \text{ or } \frac{T_1}{T_2} = \left(\frac{v_2}{v_1} \right)^{n-1}$$

$$\therefore \frac{T_1}{p_1^{(n-1)/n}} = \frac{T_2}{p_2^{(n-1)/n}} \text{ or } \frac{T_1}{T_2} = \left(\frac{p_1}{p_2} \right)^{(n-1)/n}$$

The work done in anpolytropic process for perfect gas.

$$\therefore W = \frac{p_1 v_1 - p_2 v_2}{n - 1}$$

$$\therefore W = \frac{R}{n - 1} (T_1 - T_2)$$

For mass, m,

$$\therefore W = \frac{mR}{n - 1} (T_1 - T_2)$$

Using the non-flow energy equation

$$Q = (u_2 - u_1) + W = c_v(T_2 - T_1) + \frac{R}{n-1}(T_1 - T_2)$$

$$Q = \frac{R}{n-1}(T_1 - T_2) - c_v(T_1 - T_2)$$

$$c_v = \frac{R}{\gamma - 1}$$

$$Q = \frac{R}{n-1}(T_1 - T_2) - \frac{R}{\gamma - 1}(T_1 - T_2)$$

$$i.e Q = R(T_1 - T_2) \left(\frac{1}{n-1} - \frac{1}{\gamma - 1} \right) = R(T_1 - T_2) \left(\frac{\gamma - 1 - n + 1}{(n-1)(\gamma - 1)} \right)$$

$$\therefore Q = R(T_1 - T_2) \left(\frac{\gamma - n}{(n-1)(\gamma - 1)} \right)$$

$$\therefore W = \frac{R}{n-1}(T_1 - T_2)$$

$$\therefore Q = \left(\frac{\gamma - n}{\gamma - 1} \right) \cdot W$$

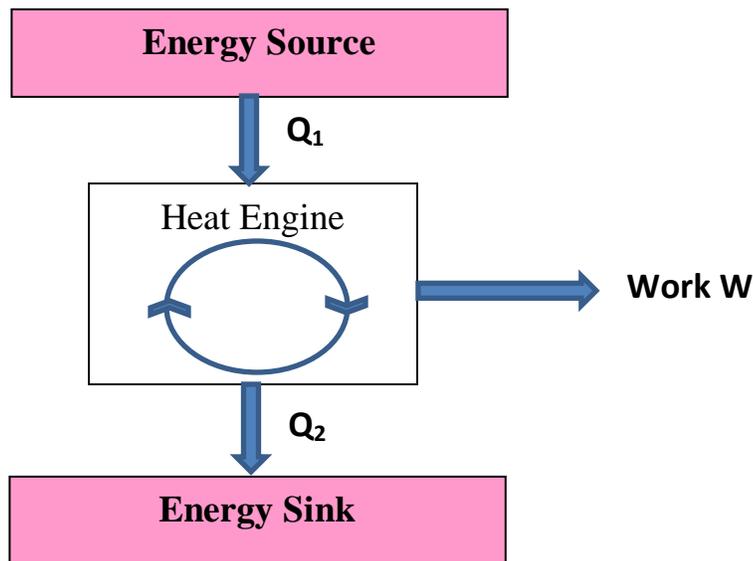
CH5

The second law of thermodynamics

The second law of thermodynamics, which is also a *natural law*, indicates that, although the net heat supplied in a cycle is equal to the net work done, **the gross heat supplied** must be *greater* than the net work done; some heat must always be rejected by the system.

5.1 The heat engine.

A heat engine is a system operating in a complete cycle and developing network from a supply of heat.



Net heat supplied = Net work done

$$\sum dQ = \sum dW$$

$$Q_1 - Q_2 = W$$

Q_1 :- is the heat supplied from the source.

Q_2 :- is the heat rejected to the sink.

W:- is the work done or by heat engine.

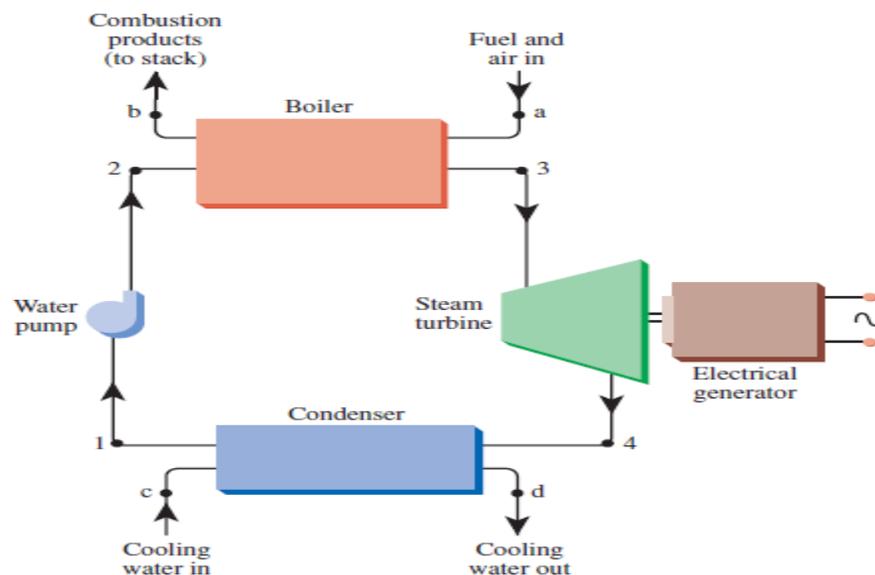
By the second law, the gross heat supplied must be greater than the net work done

$$\therefore Q_1 > W$$

The thermal efficiency of a heat engine is defined as the ratio of the net work done in the cycle to the gross heat supplied in the cycle.

$$\text{Thermal efficiency, } \eta = \frac{W}{Q_1}$$

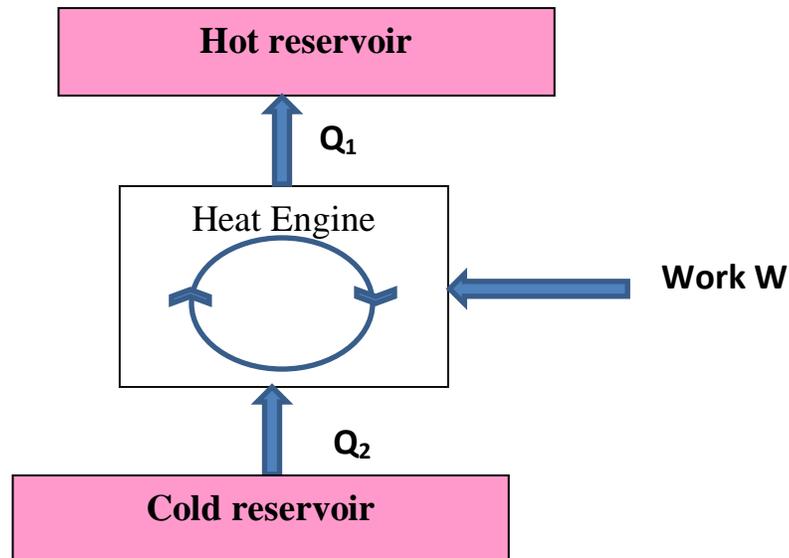
$$\text{Thermal efficiency, } \eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$



Heat is a form of energy which is transferred from one body to another body at a lower temperature.

This leads to a statement of the second law as follows:

It is impossible for a heat engine to produce network in a complete cycle if it exchanges heat only with bodies at a single fixed temperature.



$$Q_1 = Q_2 + W$$

$$W > 0$$

It is impossible to construct a device that, operating in a cycle, will produce no effect other than the transfer of heat from a cooler to a hotter body.

5.2 Entropy

Clausius realized in 1865 that he had discovered a new thermodynamic property, and he chose to name this property **entropy**. It is designated S and is defined as

$$ds = \frac{dQ}{T} \text{ (kJ/kg.K), (kJ/K)}$$

Entropy is an extensive property of a system and sometimes is referred to as *total entropy*.

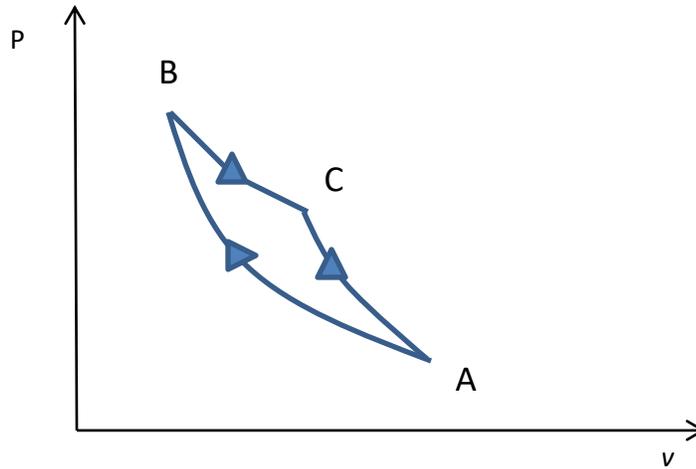


Fig. Reversible cycle on p - v diagram

Reversible adiabatic process is represented by lines AB and CA, and reversible isothermal process at temperature T_1 from B to C.

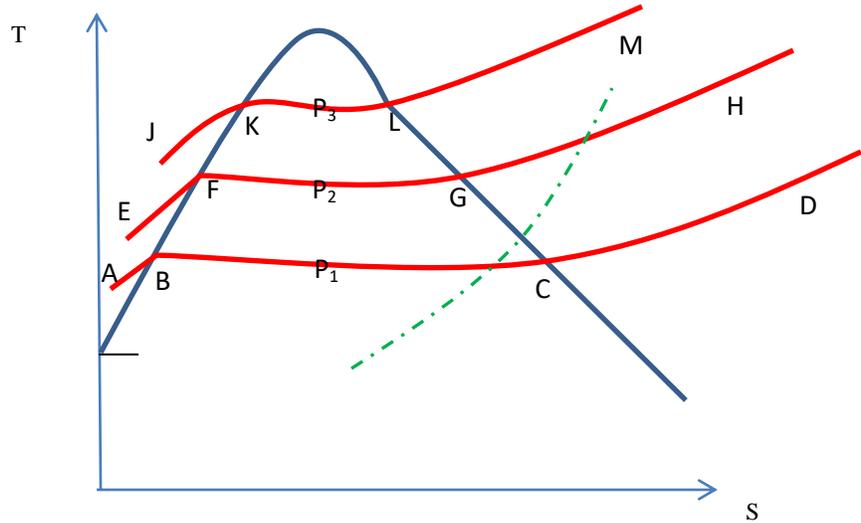
$$AB, CA, \quad Q = 0$$

$$BC, T_B = T_C = T_1$$

Hence the heat transferred is from B to C during isothermal process. The work done by system is given by enclosed area. This is impossible because it violates the second law.

5.3 The T-s diagram

(a) For a vapour



As mentioned earlier, the zero for entropy is taken as 0.01°C for steam and as -40°C for refrigerants. Three lines of constant pressure (p_1 , p_2 , p_3) are shown (lines ABCD, EFGH, and JKLM). The pressure lines in the liquid region are practically coincident with the saturated liquid lines (portions AB, EF, and jk), and the difference usually neglected. The pressure remains constant with temperature when the latent heat is added; hence the pressure lines are horizontal in the wet region (portion BC, FG, and KL). The lines curve upwards in the superheat region as shown (portions CD, GH, and LM). Thus the temperature rises as heating continues at constant pressure.

s_f is the entropy of saturated liquid.

s_g is the entropy of dry saturated vapour.

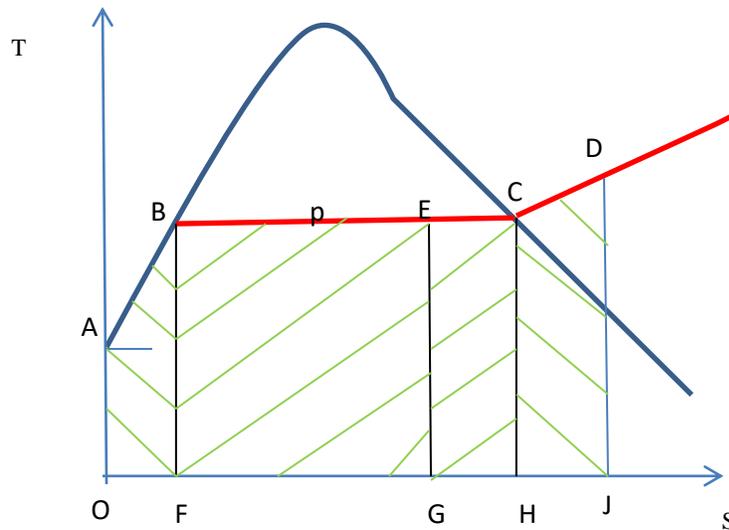
$$s_g - s_f = s_{fg}$$

$$s = (1 - x)s_f - xs_g$$

$$s = s_f - xs_{fg}$$

$$x = (s - s_f)/s_{fg}$$

$$h = h_f - xh_{fg}$$



When water at any pressure p at 0.01°C is heated at constant pressure it follows the line AB. The point b is at the saturated temperature T .

$$Q = h_B - h_A = h_B$$

$$h_A = 0 \text{ at } 0.01^\circ\text{C}$$

Therefore, we have

$$\text{Area ABFOA} = h_B = h_f$$

$$\text{Area BCHFB} = \text{Latent heat} = h_{fg} = h_c - h_B$$

Then

$$h_c = \text{Area ABFOA} + \text{Area BCHFB} = h_g$$

For wet steam at point E

$$h_E = h_B + xh_{fg}$$

$$h_E = \text{area } ABEGOA$$

The heat added from C to D at constant pressure is given by.

$$Q = h_D - h_C = \text{area } CDJHC$$

$$h_D = h_C + \text{area } CDJHC = \text{area } ABCDJOA$$

Example

1 kg of steam at 7 bar, entropy 6.5 kJ/kg K, is heated reversibly at constant pressure until the temperature is 250°C. Calculate the heat supplied, and show on a T - s diagram the area which represents the heat flow.

Solution At 7 bar, $s_g = 6.709$ kJ/kg K, hence the steam is wet, since the actual entropy, s , is less than s_g .

$$x_1 = \frac{s_1 - s_{f1}}{s_{fg1}} = \frac{6.5 - 1.992}{4.717} = 0.955$$

Then from equation (2.2)

$$h_1 = h_{f1} + x_1 h_{fg1} = 697 + (0.955 \times 2067)$$

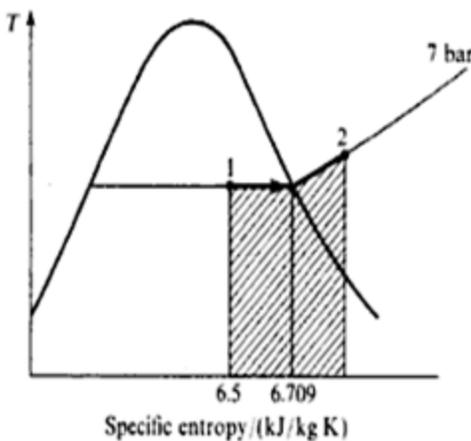
i.e. $h_1 = 697 + 1975 = 2672$ kJ/kg

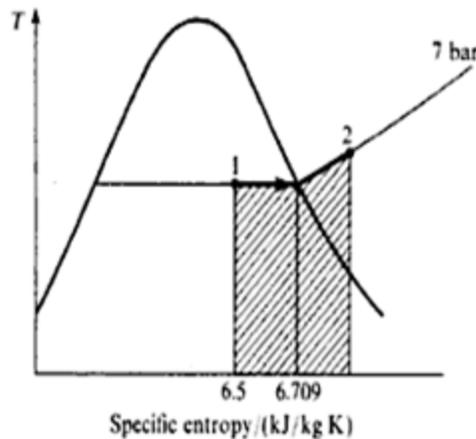
At state 2 the steam is at 250°C at 7 bar, and is therefore superheated. From superheat tables, $h_2 = 2955$ kJ/kg.

At constant pressure from equation (3.3)

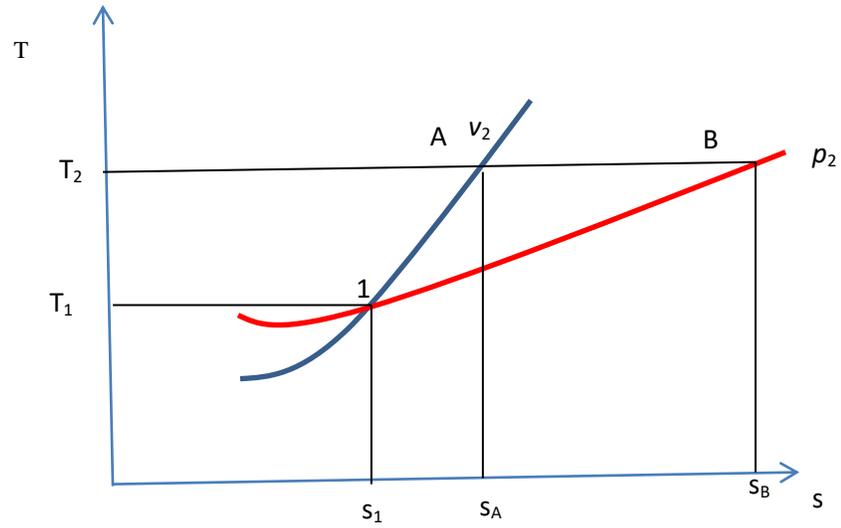
$$Q = h_2 - h_1 = 2955 - 2672 = 283$$
 kJ/kg

i.e. Heat supplied = 283 kJ/kg

The T - s diagram showing the process is given in , the shaded area representing the heat flow.



(b) For a perfect gas



$$s_A - s_1 = \int_1^A \frac{dQ}{T}$$

Also at constant volume for 1 kg of gas at from equation $dQ = c_v dT$

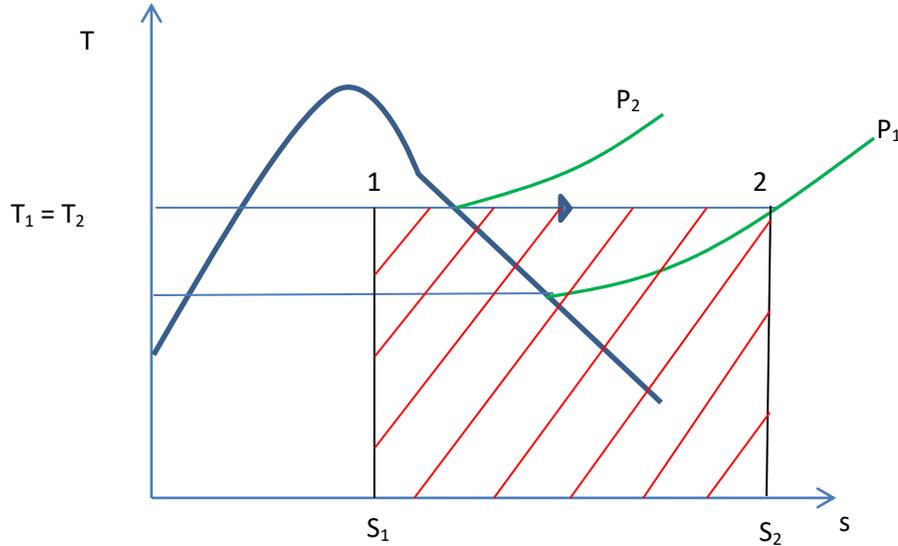
$$\therefore s_A - s_1 = \int_1^A \frac{c_v dT}{T} = c_v \ln \frac{T_A}{T_1}, (T_A = T_2)$$

Similarity, at constant pressure

$$\therefore s_B - s_1 = \int_1^B \frac{c_p dT}{T} = c_p \ln \frac{T_B}{T_1}, (T_B = T_2)$$

5.4 Reversible process on the T-s diagram

1- Reversible isothermal process



For steam

$$\text{heat supplied} = dQ = Tds, \quad Q = \int_{s_1}^{s_2} Tds = T(s_2 - s_1)$$

for perfect gas

$$dQ = du + pdv$$

$$\text{also } du = c_v dT$$

$$dQ = c_v dT + pdv, \quad \therefore dT = 0$$

$$\therefore dQ = pdv, \text{ since } pv = RT$$

$$dQ = RT \frac{dv}{v}$$

$$\therefore s_2 - s_1 = \int_1^2 \frac{dQ}{T} = \int_{v_1}^{v_2} \frac{RTdv}{Tv} = R \int_{v_1}^{v_2} \frac{dv}{v} = R \ln \frac{v_2}{v_1} = R \ln \frac{p_1}{p_2}$$

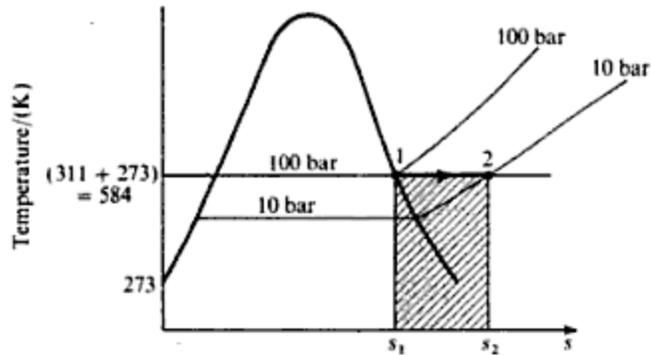
$$\text{therefor } Q = T(s_2 - s_1) = RT \ln \frac{v_2}{v_1} = RT \ln \frac{p_1}{p_2}$$

$$\text{therefor } Q = w = T(s_2 - s_1) = RT \ln \frac{v_2}{v_1} = RT \ln \frac{p_1}{p_2} = p_1 v_1 \ln \frac{p_1}{p_2}$$

Example

Dry saturated steam at 100 bar expands isothermally and reversibly to a pressure of 10 bar. Calculate the heat supplied and the work done per kilogram of steam during the process.

Solution The process is shown in Fig. 4.15, the shaded area representing the heat supplied.



From tables at 100 bar, dry saturated

$$s_1 = s_g = 5.615 \text{ kJ/kg K} \quad \text{and} \quad t_1 = 311^\circ\text{C}$$

At 10 bar and 311°C the steam is superheated, hence interpolating

$$s_2 = 7.124 + \left(\frac{311 - 300}{350 - 300} \right) (7.301 - 7.124)$$

$$\text{i.e.} \quad s_2 = 7.124 + 0.039 = 7.163 \text{ kJ/kg K}$$

Then we have

$$\begin{aligned} \text{Heat supplied} &= \text{shaded area} = T(s_2 - s_1) \\ &= 584(7.163 - 5.615) = 584 \times 1.548 \end{aligned}$$

where $T = 311 + 273 = 584 \text{ K}$.

$$\text{i.e.} \quad \text{Heat supplied} = 904 \text{ kJ/kg}$$

To find the work done it is necessary to apply the non-flow energy equation,

$$\text{i.e.} \quad Q + W = u_2 - u_1 \quad \text{or} \quad W = (u_2 - u_1) - Q$$

From tables, at 100 bar, dry saturated,

$$u_1 = u_g = 2545 \text{ kJ/kg}$$

At 10 bar and 311°C , interpolating,

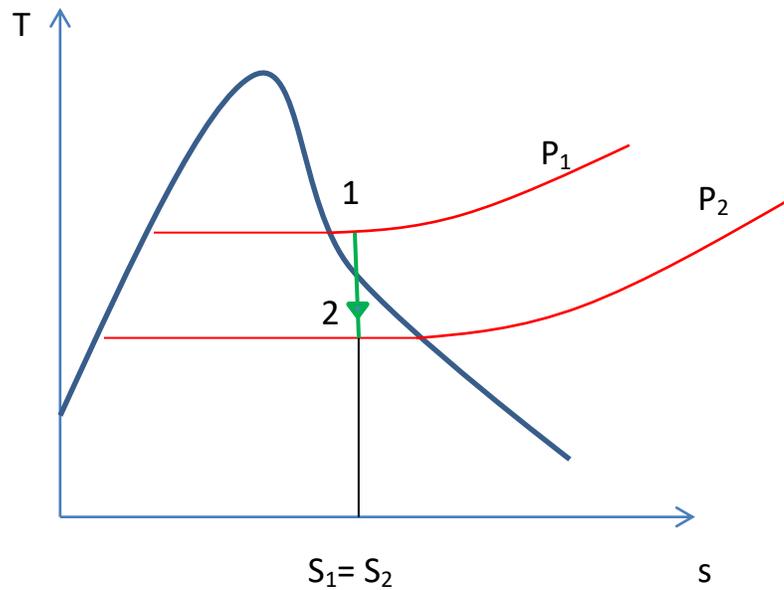
$$u_2 = 2794 + \left(\frac{311 - 300}{350 - 300} \right) (2875 - 2794) = 2794 + 17.8$$

$$\text{i.e.} \quad u_2 = 2811.8 \text{ kJ/kg}$$

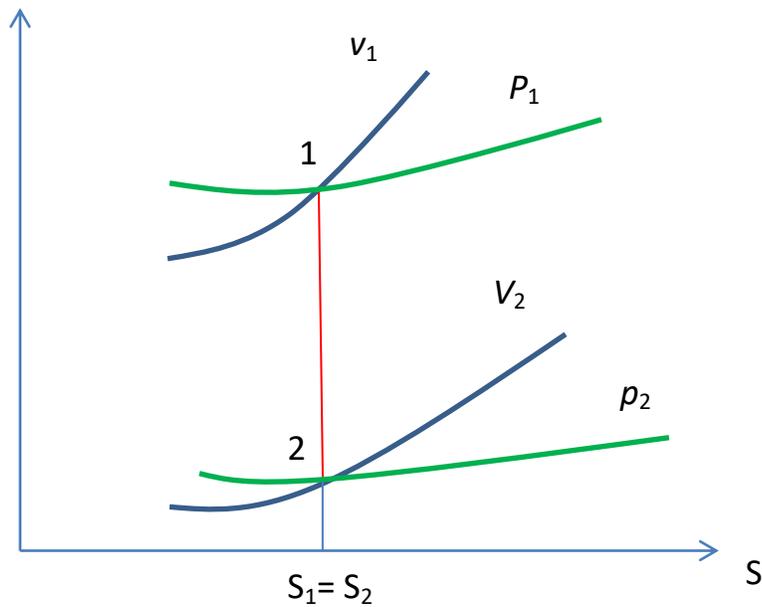
2- Reversible adiabatic process (isentropic process)

For a reversible adiabatic process, the entropy remains constant ($s=C.$, $Q=0$)

For steam



For perfect gas



for a perfect gas the process follows a law $pv^\gamma = \text{constant}$

Example Steam at 100 bar, 375 °C expands isentropically in a cylinder behind a piston to a pressure of 10 bar. Calculate the work done per kilogram of steam.

Solution From superheat tables, at 100 bar, 375 °C, we have

$$s_1 = s_2 = 6.091 \text{ kJ/kg K}$$

At 10 bar and $s_2 = 6.091$, the steam is wet, since s_2 is less than s_{g_2} . Then from equation (4.12)

$$x_2 = \frac{s_2 - s_{f_2}}{s_{fg_2}} = \frac{6.091 - 2.138}{4.448} = 0.889$$

Then from equation (2.3)

$$u_1 = (1 - x_2)u_{f_2} + x_2u_{g_2} = (0.111 \times 762) + (0.889 \times 2584)$$

i.e. $u_2 = 84.6 + 2297 = 2381.6 \text{ kJ/kg}$

At 100 bar, 375 °C, we have from tables, $h_1 = 3017 \text{ kJ/kg}$ and $v_1 = 0.02453 \text{ m}^3/\text{kg}$. Then using equation (1.9)

$$u_1 = h_1 - p_1 v_1 = 3017 - \frac{100 \times 10^5 \times 0.02453}{10^3} = 3017 - 245.3$$

i.e. $u_1 = 2771.7 \text{ kJ/kg}$

For an adiabatic process from equation (3.13),

$$W = u_2 - u_1$$

therefore,

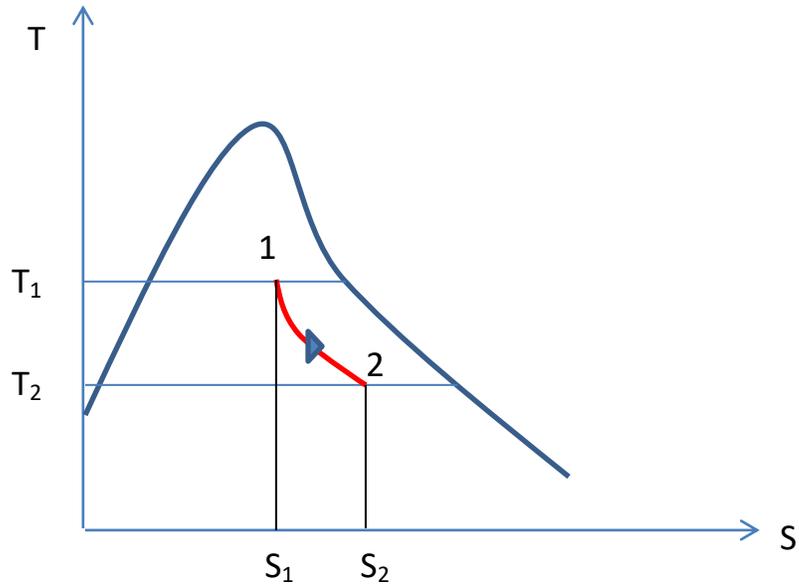
$$W = 2381.6 - 2771.7$$

$$= -390.1 \text{ kJ/kg}$$

i.e. Work done by the steam = 390.1 kJ/kg

3- Polytropic process

To find the change of entropy in a polytropic process for a vapour when the end states have been fixed using $p_1 v_1^n = p_2 v_2^n$ the entropy values at the end states can be read straight from tables.



For perfect gas

The non-flow energy equation.

$$dQ = du + pdv$$

From Joules law $du = c_v dT$ and from equation $pv = RT$

$$dQ = c_v dT + RT \frac{dv}{v}$$

$$\text{then } ds = \frac{dQ}{T} = c_v \frac{dT}{T} + R \frac{dv}{v}$$

Hence between any two states 1 and 2

$$\therefore s_2 - s_1 = c_v \int_{T_1}^{T_2} \frac{dT}{T} + \int_{v_1}^{v_2} \frac{Rdv}{v} = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

At constant temperature between p_2 and p_1

$$\therefore s_B - s_1 = R \ln \frac{p_1}{p_2}$$

And at constant pressure between T_2 and T_1 we have

$$s_B - s_2 = c_p \ln \frac{T_1}{T_2}$$

Hence

$$s_2 - s_1 = R \ln \frac{p_1}{p_2} - c_p \ln \frac{T_1}{T_2}$$

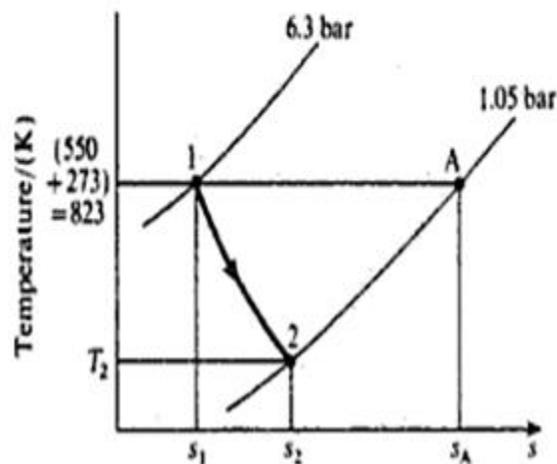
Or

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} + R \ln \frac{p_1}{p_2}$$

Example Calculate the change of entropy of 1 kg of air expanding polytropically in a cylinder behind a piston from 6.3 bar and 550°C to 1.05 bar. The index of expansion is 1.3.

Solution The process is shown on a T - s diagram in Fig. 4.22. From equation 3.29,

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2} \right)^{(n-1)/n} = \left(\frac{6.3}{1.05} \right)^{(1.3-1)/1.3} = 1.512$$



therefore

$$T_2 = \frac{823}{1.512} = 544 \text{ K}$$

where $T_1 = 550 + 273 = 823 \text{ K}$.

Now replace the process 1 to 2 by two processes, 1 to A and A to 2. Then at constant temperature from 1 to A, from equation (4.13)

$$\begin{aligned} s_A - s_1 &= R \ln\left(\frac{p_1}{p_2}\right) = 0.287 \ln\left(\frac{6.3}{1.05}\right) \\ &= 0.514 \text{ kJ/kg K} \end{aligned}$$

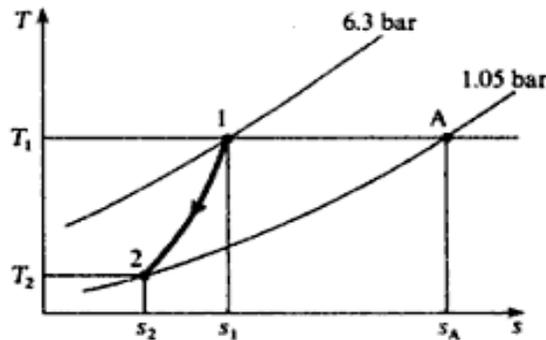
At constant pressure from A to 2

$$\begin{aligned} s_A - s_2 &= c_p \ln\left(\frac{T_1}{T_2}\right) = 1.005 \ln\left(\frac{823}{544}\right) \\ &= 0.416 \end{aligned}$$

Then $s_2 - s_1 = 0.514 - 0.416 = 0.098 \text{ kJ/kg K}$

i.e. Increase in entropy = 0.098 kJ/kg K

Note that if in Example $s_A - s_2$ happened to be greater than $s_A - s_1$, this would mean that s_1 was greater than s_2 , and the process should appear as in Fig.



4.5 Entropy and irreversibility

In section 4.4 it is pointed out that, since entropy is a property, the change of entropy depends only on the end states and not on the process between the end states. Therefore, provided an irreversible process gives enough information to fix the end states then the change of entropy can be found. This can best be illustrated by some examples.

Example Steam at 7 bar, dryness fraction 0.96, is throttled down to 3.5 bar. Calculate the change of entropy per unit mass of steam.

Solution At 7 bar, dryness fraction 0.96, we have

$$s_1 = s_{f_1} + x_1 s_{fg_1} = 1.992 + (0.96 \times 4.717)$$

i.e. $s_1 = 6.522 \text{ kJ/kg K}$

In section 3.4 it is shown that for a throttling process, $h_1 = h_2$.

$$h_2 = h_1 = h_{f_1} + x_1 h_{fg_1} = 697 + (0.96 \times 2067) = 2682 \text{ kJ/kg}$$

At 3.5 bar and $h_2 = 2682 \text{ kJ/kg}$ the steam is still wet, since $h_{g_2} > h_2$.

$h_2 = h_{f_2} + x_2 h_{fg_2}$, therefore

$$x_2 = \frac{h_2 - h_{f_2}}{h_{fg_2}} = \frac{2682 - 584}{2148} = 0.977$$

Then

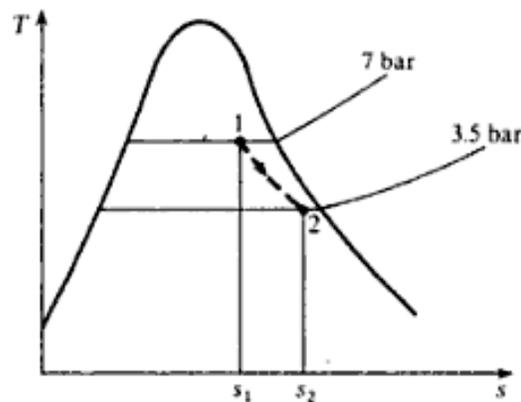
$$s_2 = s_{f_2} + x_2 s_{fg_2} = 1.727 + (0.977 \times 5.214) = 6.817 \text{ kJ/kg K}$$

Therefore,

$$\text{Increase of entropy} = 6.817 - 6.522 = 0.295 \text{ kJ/kg K}$$

The process is shown on a T - s diagram in Fig. Note that the process is shown dotted, and the area under the line does not represent heat flow; a throttling process assumes no heat flow, but there is a change in entropy because the process is irreversible.

Fig. Throttling process for Exa on a T - s diagram



Example Two vessels of equal volume are connected by a short length of pipe containing a valve; both vessels are well lagged. One vessel contains air and the other is completely evacuated. Calculate the change of entropy per kg of air in the system when the valve is opened and the air is allowed to fill both vessels.

Solution Initially the vessel A contains air and the vessel B is completely evacuated, as in Fig. ; finally the air occupies both vessels A and B. In section 4 it was shown that in an unresisted expansion for a perfect gas, the initial and final temperatures are equal. In this case the initial volume is V_A and the final volume

Fig. Two well-lagged interconnected vessels for Example

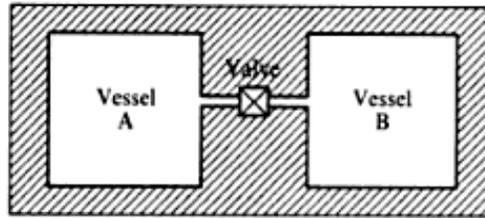
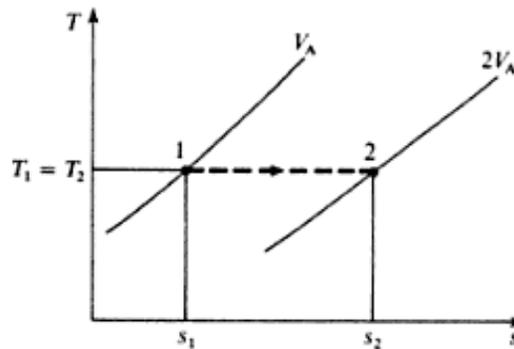


Fig. Process on a $T-s$ diagram for Example



is $V_A + V_B = 2V_A$. The end states can be marked on a $T-s$ diagram as shown in Fig. The process 1 to 2 is irreversible and must be drawn dotted. The change of entropy is $s_2 - s_1$, regardless of the path of the process between states 1 and 2. Hence, for the purpose of calculating the change of entropy, imagine the process replaced by a reversible isothermal process between states 1 and 2. Then

$$\begin{aligned}(s_2 - s_1) &= R \ln\left(\frac{V_2}{V_1}\right) = 0.287 \ln\left(\frac{2V_A}{V_A}\right) \\ &= 0.287 \ln 2 = 0.199 \text{ kJ/kg K}\end{aligned}$$

i.e. Increase of entropy = 0.199 kJ/kg K

Note that the process is drawn dotted in Fig. , and the area under the line has no significance; the process is adiabatic and there is a change in entropy since the process is irreversible.

It is important to remember that the equation $ds = dQ/T$, is true only for reversible processes. In the same way the equation $dW = -p dv$ is true only for reversible processes. In Example the volume of the air increased from V_A to $2V_A$, and yet no work was done by the air during the process,

i.e. $dW = 0$ yet $V_2 - V_1 = 2V_A - V_A = V_A$

Similarly, the entropy in Example increased by 0.199 kJ/kg K and yet the heat flow was zero, i.e. $ds \neq dQ/T$. No confusion should be caused if the T - s and/or the p - v diagram is drawn for each problem and the state points marked in their correct positions. Then, when a process between two states is reversible, the lines representing the process can be drawn in as full lines, and the area under the line represents heat flow on the T - s diagram and work done on the p - v diagram. When the process between the states is irreversible the line must be drawn dotted, and the area under the line has no significance on either diagram.

It can be shown from the second law that the entropy of a thermally isolated system must either increase or remain the same. For instance, a system undergoing an adiabatic process is thermally isolated from its surroundings since no heat flows to or from the system. We have seen that in a reversible adiabatic process the entropy remains the same. In an irreversible adiabatic process the entropy must always increase, and the gain of entropy is a measure of the irreversibility of the process. The processes in Examples illustrate this fact. As another example, consider an irreversible adiabatic expansion in a steam turbine as shown in Fig. as process 1 to 2. A reversible adiabatic process between the same pressures is represented by 1 to 2s in Fig. The increase of entropy, $s_2 - s_1 = s_2 - s_{2s}$, is a measure of the irreversibility of the process. Similarly, in Fig. 4.29, an irreversible adiabatic compression in a rotary compressor is shown as process 1 to 2. A reversible adiabatic process between the same pressures is represented by 1 to 2s. As before, the increase of entropy shows the irreversibility of the process.

Fig. Irreversible adiabatic process for steam on a $T-s$ diagram

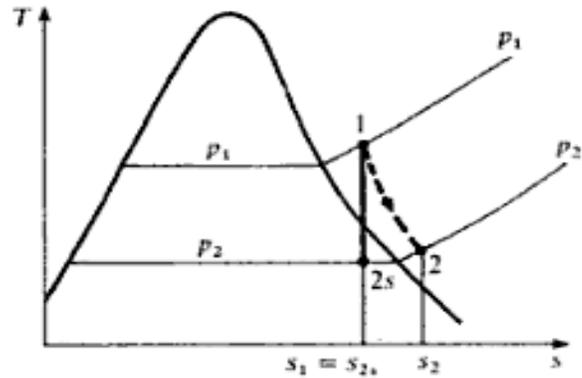
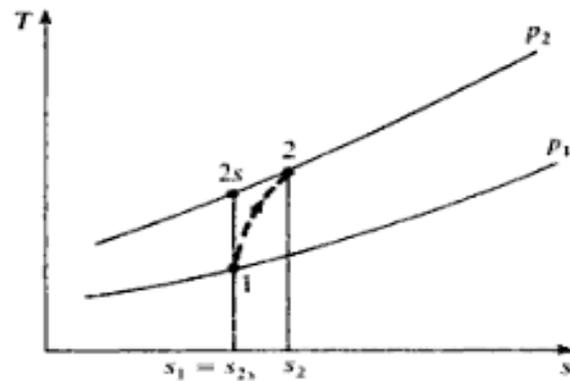


Fig. Irreversible adiabatic compression for a perfect gas on a $T-s$ diagram



Example In an air turbine the air expands from 6.8 bar and 430°C to 1.013 bar and 150°C . The heat loss from the turbine can be assumed to be negligible. Show that the process is irreversible, and calculate the change of entropy per kilogram of air.

Solution Since the heat loss is negligible the process is adiabatic. For a reversible adiabatic process for a perfect gas,

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{(\gamma-1)/\gamma}$$

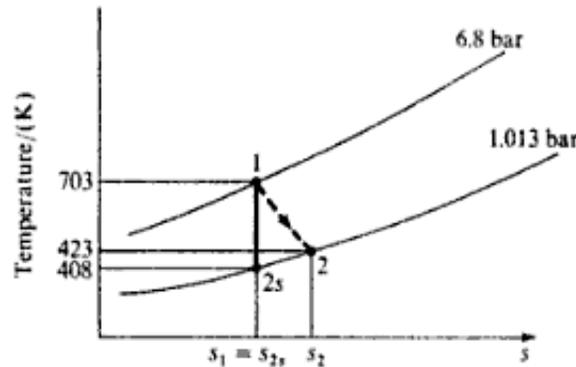
i.e.
$$\frac{703}{T_2} = \left(\frac{6.8}{1.013}\right)^{(1.4-1)/1.4}$$

where $T_1 = 430 + 273 = 703 \text{ K}$,

$$T_2 = \frac{703}{1.723} = 408 \text{ K} = 408 - 273 = 135^\circ\text{C}$$

But the actual temperature is 150°C at the pressure of 1.013 bar, hence the process is irreversible. The process is shown as 1 to 2 in Fig. ; the ideal isentropic process 1 to 2s is also shown. It is not possible for process 1 to 2 to be reversible, because in that case the area under line 1–2 would represent heat flow and yet the process is adiabatic.

Fig. *T-s* diagram for Example



The change of entropy, $s_2 - s_1$, can be found by considering a reversible constant pressure process between 2s and 2. Then from equation $ds = dQ/T$, and at constant pressure for 1 kg of a perfect gas we have $dQ = c_p dT$ therefore

$$\begin{aligned} s_2 - s_{2s} &= \int_{2s}^2 \frac{c_p dT}{T} = c_p \ln\left(\frac{T_2}{T_{2s}}\right) \\ &= 1.005 \ln\left(\frac{423}{408}\right) = 0.0363 \text{ kJ/kg K} \end{aligned}$$

i.e. Increase of entropy, $s_2 - s_1 = 0.0363 \text{ kJ/kg K}$

Consider now the case when a system is not thermally isolated from its surroundings. The entropy of such a system can increase, decrease, or remain the same, depending on the heat crossing the boundary. However, if the boundary is extended to include the source or sink of heat with which the system is in communication, then the entropy of this new system must either increase or remain the same. To illustrate this, consider a hot reservoir at T_1 and a cold

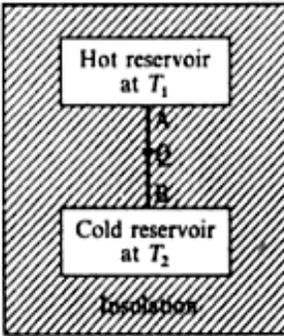


Fig. Two thermally-insulated interconnected reservoirs of energy

reservoir at T_2 , and assume that the two reservoirs are thermally insulated from the surroundings as in Fig. Let the heat flow from the hot to the cold reservoir be Q . There is a continuous temperature gradient from T_1 to T_2 between points A and B, and it can be assumed that heat is transferred reversibly from the hot reservoir to point A, and from point B to the cold reservoir. It will be assumed that the reservoirs are such that the temperature of each remains constant. Then we have

$$\text{Heat supplied to cold reservoir} = +Q$$

Hence from equation

$$\text{Increase of entropy of cold reservoir} = + \frac{Q}{T_2}$$

Also,

$$\text{Heat supplied to hot reservoir} = -Q$$

therefore

$$\text{Increase of entropy of hot reservoir} = - \frac{Q}{T_1}$$

$$\text{i.e. Net increase of entropy of system, } \Delta s = \left(\frac{Q}{T_2} - \frac{Q}{T_1} \right)$$

Since $T_1 > T_2$ it can be seen that Δs is positive, and hence the entropy of the system must increase. In the limit, when the difference in temperature is infinitely small, then $\Delta s = 0$. This confirms the principle that the entropy of an isolated system must either increase or remain the same. In section 1., criterion (c) for reversibility was stated as follows: the difference in temperature between a system and its surroundings must be infinitely small during a reversible process.

In the above example, when $T_1 > T_2$, then the heat flow between the reservoirs is irreversible by the above criterion. Thus the entropy of the system increases when the heat flow process is irreversible, but remains the same when the process is reversible. The increase of entropy is a measure of the irreversibility. The processes occurring in the above example can be drawn on a T - s diagram as in Fig. The two processes have been superimposed on the same diagram.

Process P-R represents the transfer of Q units of heat from the hot reservoir,

Ch. 6

The Heat Engine Cycles

6.1 THE CARNOT CYCLE

We mentioned earlier that heat engines are cyclic devices and that the working fluid of a heat engine returns to its initial state at the end of each cycle. Work is done by the working fluid during one part of the cycle and on the working fluid during another part. The difference between these two is the network delivered by the heat engine. The efficiency of a heat-engine cycle greatly depends on how the individual processes that make up the cycle are executed. The network, thus the cycle efficiency, can be maximized by using processes that require the least amount of work and deliver the most, that is, by using reversible processes. Therefore, it is no surprise that the most efficient cycles are reversible cycles, that is, cycles that consist entirely of reversible processes. Reversible cycles cannot be achieved in practice because the irreversibilities associated with each process cannot be eliminated. However, reversible cycles provide upper limits on the performance of real cycles. Heat engines and refrigerators that work on reversible cycles serve as models to which actual heat engines and refrigerators can be compared. Reversible cycles also serve as starting points in the development of actual cycles and are modified as needed to meet certain requirements.

Probably the best known reversible cycle is the Carnot cycle, first proposed in 1824 by French engineer Sadi Carnot. The theoretical heat engine that operates on the Carnot cycle is called the Carnot heat engine. The Carnot cycle is composed of four reversible processes—two isothermal and two adiabatic—and it can be executed either in a closed or a steady-flow system.

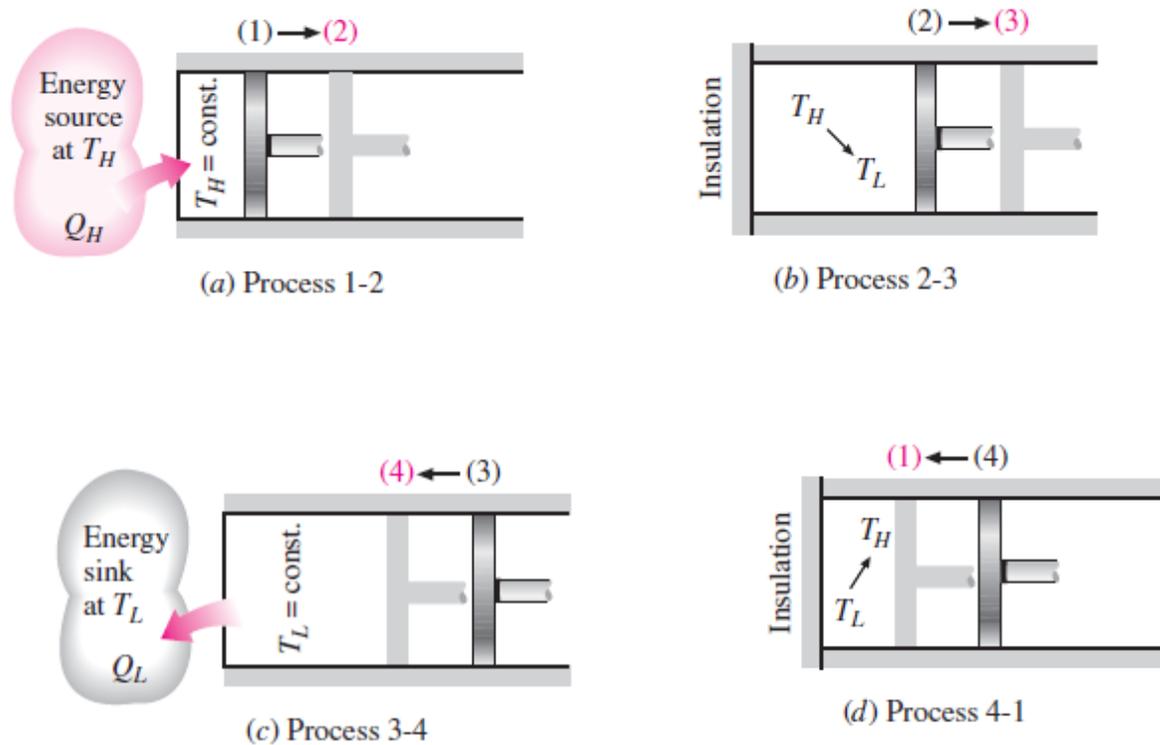


Fig 6.1 Execution of the Carnot cycle in a closed system.

- 1- **Reversible Isothermal Expansion** (process 1-2, $T_H = \text{constant}$). Initially (State 1), the temperature of the gas is T_H and the cylinder head is in close contact with a source at temperature T_H .
- 2- **Reversible Adiabatic Expansion** (process 2-3, temperature drops from T_H to T_L). At state 2, the reservoir that was in contact with the cylinder head is removed and replaced by insulation so that the system becomes adiabatic.
- 3- **Reversible Isothermal Compression** (process 3-4, $T_L = \text{constant}$). At state 3, the insulation at the cylinder head is removed, and the cylinder is brought into contact with a sink at temperature T_L .
- 4- **Reversible Adiabatic Compression** (process 4-1, temperature rises from T_L to T_H).

The thermal efficiency of any heat engine, reversible or irreversible, is given by

$$\text{Thermal efficiency, } \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

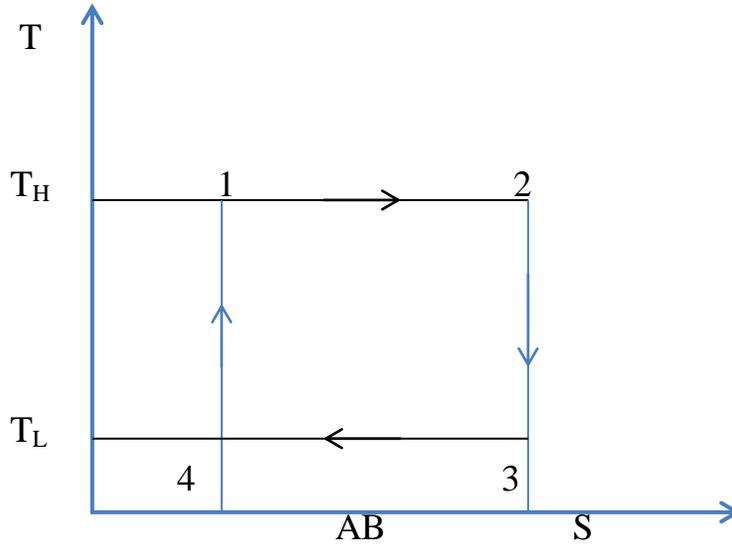


Fig. 6.2 T-S diagram of the Carnot cycle.

$$Q_1 = \text{area } 12BA1 = T_H(s_B - s_A)$$

$$Q_2 = \text{area } 34AB3 = T_L(s_B - s_A)$$

$$\text{Thermal efficiency of carnot cycle, } \eta_{carnot} = 1 - \frac{T_L(s_B - s_A)}{T_H(s_B - s_A)}$$

$$\eta_{carnot} = 1 - \frac{T_L}{T_H}$$

The work output of the Carnot cycle from the T-s diagram

$$\sum Q = \sum W$$

Therefore, the work output of the cycle is given by,

$$W = Q_1 - Q_2$$

$$W_{carnot} = \text{area } 12341 = (T_H - T_L)(s_B - s_A)$$

The P - V diagram of this cycle is shown in Fig. 6.3. Remembering that on a P - V diagram the area under the process curve represents the boundary work for quasi-equilibrium (internally reversible) processes, we see that the area under curve 1-2-3 is the work done by the gas during the expansion part of the cycle, and the area under curve 3-4-1 is the work done on the gas during the compression part of the cycle. The area enclosed by the path of the cycle (area 1-2-3-4-1) is the difference between these two and represents the net work done during the cycle.

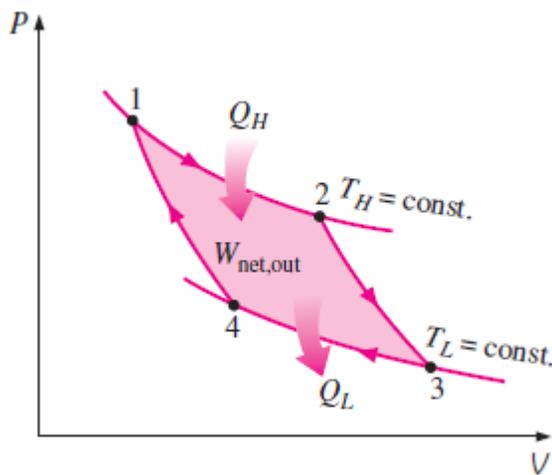


Fig. 6.3 P - V diagram of the Carnot cycle.

The ratio of the network output to the gross work output of the system is called the work ratio.

6.2 Absolute temperature scale

Using the second law of thermodynamics it is possible to establish a temperature scale which is independent of the working substance.

We have, for any heat engine

$$\text{Thermal efficiency, } \eta = 1 - \frac{Q_2}{Q_1}$$

Also the efficiency of an engine operating on the Carnot cycle depends only on the temperature of the hot and cold reservoirs. Denoting temperature on any arbitrary scale by X, we have

$$\eta = \phi (X_1, X_2)$$

Where ϕ is a function, X_1 and X_2 are the temperatures of the hot and cold reservoirs.

$$\frac{Q_2}{Q_1} = F(X_1, X_2)$$

Where F is a new function

$$\frac{Q_2}{Q_1} = \frac{X_2}{X_1}$$

Also

$$\eta = 1 - \frac{T_2}{T_1}$$

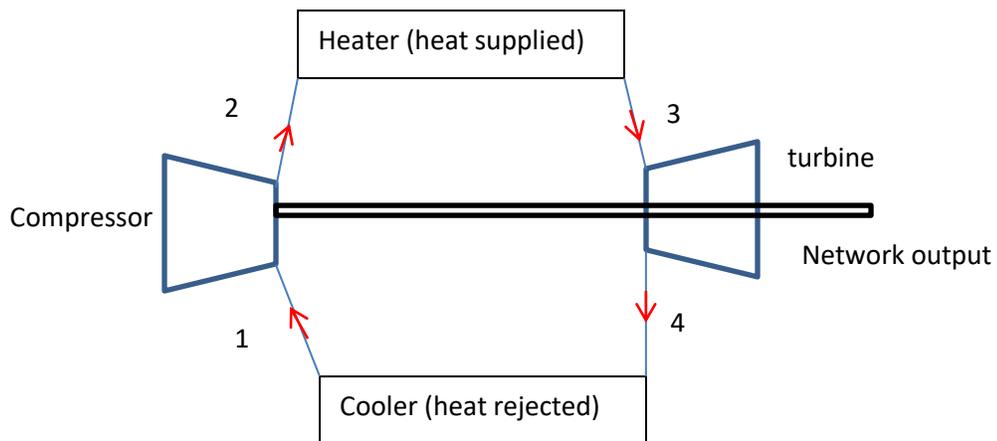
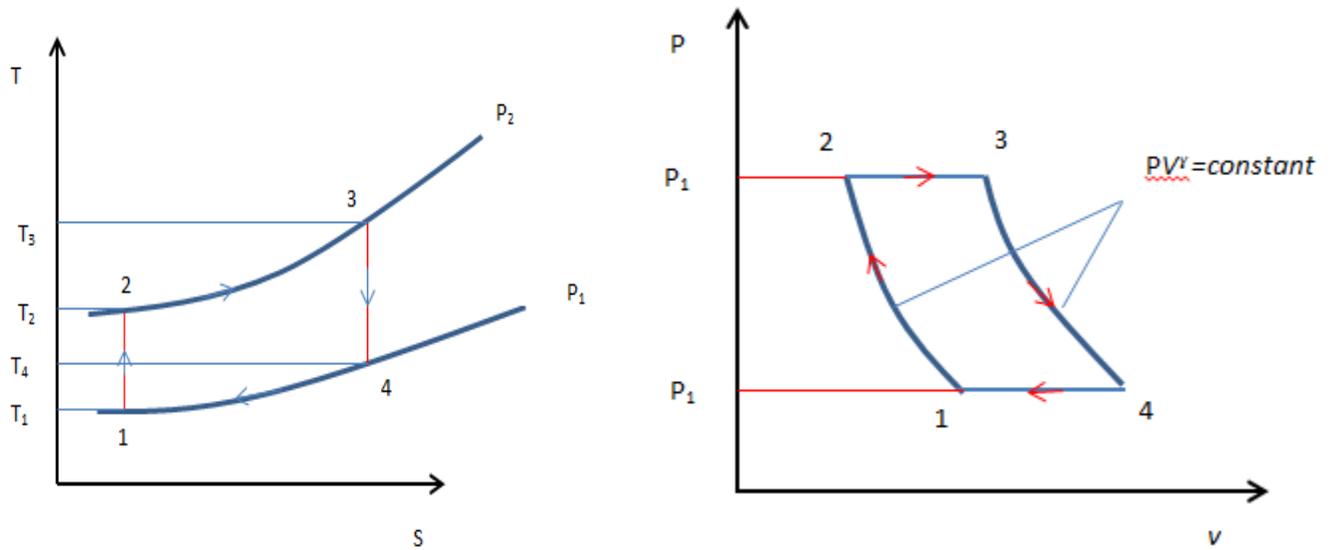
$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

Or

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

6.3 the constant pressure cycle (Joule or Brayton Cycle)

In this cycle the heat supply and heat rejected processes occur reversibly at constant pressure. The expansion and compression processes are isentropic.



Work input to compressor = $(h_2 - h_1) = c_p(T_2 - T_1)$

Work output from turbine = $(h_3 - h_4) = c_p(T_3 - T_4)$

Heat supplied in heater, $Q_1 = (h_3 - h_2) = c_p(T_3 - T_2)$

Heat rejected in cooler, $Q_2 = (h_4 - h_1) = c_p(T_4 - T_1)$

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{c_p (T_4 - T_1)}{C_p (T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Now since process 1 to 2 and 3 to 4 are isentropic between the same pressure P_2, P_1 , we have using equation

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = \frac{T_3}{T_4} = r_p^{(\gamma-1)/\gamma}$$

Where the r_p is the pressure ratio

Then

$$T_3 = T_4 r_p^{(\gamma-1)/\gamma}$$

$$T_2 = T_1 r_p^{(\gamma-1)/\gamma}$$

$$\therefore (T_3 - T_2) = (T_4 - T_1) r_p^{(\gamma-1)/\gamma}$$

Hence

$$\eta = 1 - (T_4 - T_1) / (T_4 - T_1) r_p^{(\gamma-1)/\gamma}$$

$$\eta = 1 - 1 / r_p^{(\gamma-1)/\gamma}$$

The work ratio of the constant pressure cycle may be found as follows

$$\text{work ratio} = \frac{\text{Network}}{\text{Grosswork}} = \frac{c_p ((T_3 - T_4) - (T_2 - T_1))}{C_p (T_3 - T_4)} = 1 - \frac{T_2 - T_1}{T_3 - T_4}$$

$$T_2 = T_1 r_p^{(\gamma-1)/\gamma}$$

$$T_4 = T_3 / r_p^{(\gamma-1)/\gamma}$$

Hence

$$\text{work ratio} = 1 - (T_1 (r_p^{\frac{\gamma-1}{\gamma}} - 1)) / (T_3 (1 - (\frac{1}{r_p^{(\gamma-1)/\gamma}})))$$

6.4 AIR-STANDARD ASSUMPTIONS

In gas power cycles, the working fluid remains a gas throughout the entire cycle. Spark-ignition engines, diesel engines, and conventional gas turbines are familiar examples of devices that operate on gas cycles. In all these engines, energy is provided by burning a fuel within the system boundaries. That is, they are internal combustion engines. Because of this combustion process, the composition of the working fluid changes from air and fuel to combustion products during the course of the cycle. However, considering that air is predominantly nitrogen that undergoes hardly any chemical reactions in the combustion chamber, the working fluid closely resembles air at all times.

Even though internal combustion engines operate on a mechanical cycle (the piston returns to its starting position at the end of each revolution), the working fluid does not undergo a complete thermodynamic cycle. It is thrown out of the engine at some point in the cycle (as exhaust gases) instead of being returned to the initial state. Working on an open cycle is the characteristic of all internal combustion engines.

The actual gas power cycles are rather complex. To reduce the analysis to a manageable level, we utilize the following approximations, commonly known as the air-standard assumptions:

- 1-The working fluid is air, which continuously circulates in a closed loop and always behaves as an ideal gas.
- 2-All the processes that make up the cycle are internally reversible.
- 3-The combustion process is replaced by a heat-addition process from an external source (Fig. 6.5).
- 4-The exhaust process is replaced by a heat-rejection process that restores the working fluid to its initial state.

Another assumption that is often utilized to simplify the analysis even more is that air has constant specific heats whose values are determined at room temperature (25°C, or 77°F). When this assumption is utilized, the air-standard assumptions are called the cold-air-standard assumptions. A cycle for which the air-standard assumptions are applicable is frequently referred to as an air-standard cycle.

The air-standard assumptions previously stated provide considerable simplification in the analysis without significantly deviating from the actual cycles. This simplified model enables us to study qualitatively the influence of major parameters on the performance of the actual engines.

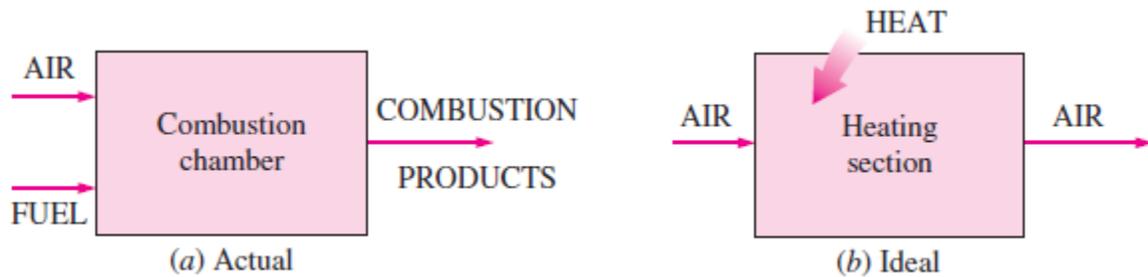


Fig. 6.5 The combustion process is replaced by a heat-addition process in ideal cycles.

6.5 AN OVERVIEW OF RECIPROCATING ENGINES

Despite its simplicity, the reciprocating engine (basically a piston–cylinder device) is one of the rare inventions that have proved to be very versatile and to have a wide range of applications. It is the powerhouse of the vast majority of automobiles, trucks, light aircraft, ships, and electric power generators, as well as many other devices. The basic components of a reciprocating engine are shown in Fig. 6.6. The piston reciprocates in the cylinder between two fixed positions called the top dead center (TDC)—the position of the piston when it forms the smallest volume in the cylinder—and the bottom dead center (BDC)—the position of the piston when it forms the largest volume in the cylinder. The distance between the TDC and the BDC is the largest distance that the piston can travel in one direction, and it is called the stroke of the engine. The diameter of the piston is called the bore. The air or air–fuel mixture is drawn into the cylinder through the intake valve, and the combustion products are expelled from the cylinder through the exhaust valve. The minimum volume formed in the cylinder when the piston is at TDC is called the clearance volume (Fig. 6.7). The volume displaced by the piston as it moves between TDC and BDC is called the displacement volume. The ratio of the maximum volume formed in the cylinder to the minimum (clearance) volume is called the compression ratio r_v of the engine:

$$r_v = \frac{v_{max}}{v_{min}} = \frac{v_{BDC}}{v_{TDC}}$$

Notice that the compression ratio is a volume ratio and should not be confused with the pressure ratio.

Another term frequently used in conjunction with reciprocating engines is the mean effective pressure (MEP). It is a fictitious pressure that, if it acted on the piston during the entire power stroke, would produce the same amount of network as that produced during the actual cycle. That is,

$$W_{net} = MEP * Piston\ area * Stroke$$

$$= MEP * Displacement\ volume$$

or

$$MEP = \frac{W_{net}}{v_{max} - v_{min}} \text{ (kpa)}$$

The mean effective pressure can be used as a parameter to compare the performances of reciprocating engines of equal size. The engine with a larger value of MEP delivers more network per cycle and thus performs better.

Reciprocating engines are classified as spark-ignition (SI) engines or compression-ignition (CI) engines, depending on how the combustion process in the cylinder is initiated. In SI engines, the combustion of the air–fuel mixture is initiated by a spark plug. In CI engines, the air–fuel mixture is self-ignited as a result of compressing the mixture above its self ignition temperature. In the next two sections, we discuss the Otto and Diesel cycles, which are the ideal cycles for the SI and CI reciprocating engines, respectively.

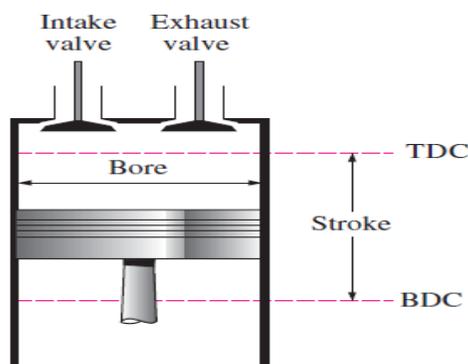


Fig. 6.6

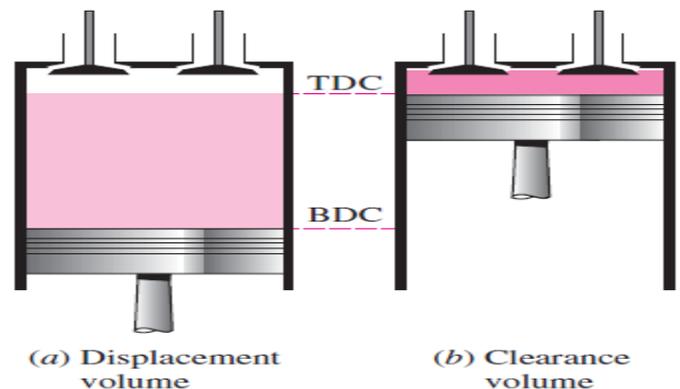


Fig. 6.7

6.6 OTTO CYCLE: THE IDEAL CYCLE FOR SPARK-IGNITION ENGINES

The Otto cycle is the ideal cycle for spark-ignition reciprocating engines. It is named after Nikolaus A. Otto, who built a successful four-stroke engine in 1876 in Germany using the cycle proposed by Frenchman Beau de Rochas in 1862. In most spark-ignition engines, the piston executes four complete strokes (two mechanical cycles) within the cylinder, and the crankshaft completes two revolutions for each thermodynamic cycle. These engines are called **four-stroke** internal combustion engines. A schematic of each stroke as well as a P - v diagram for an actual four-stroke spark-ignition engine is given in Fig. 6.8.

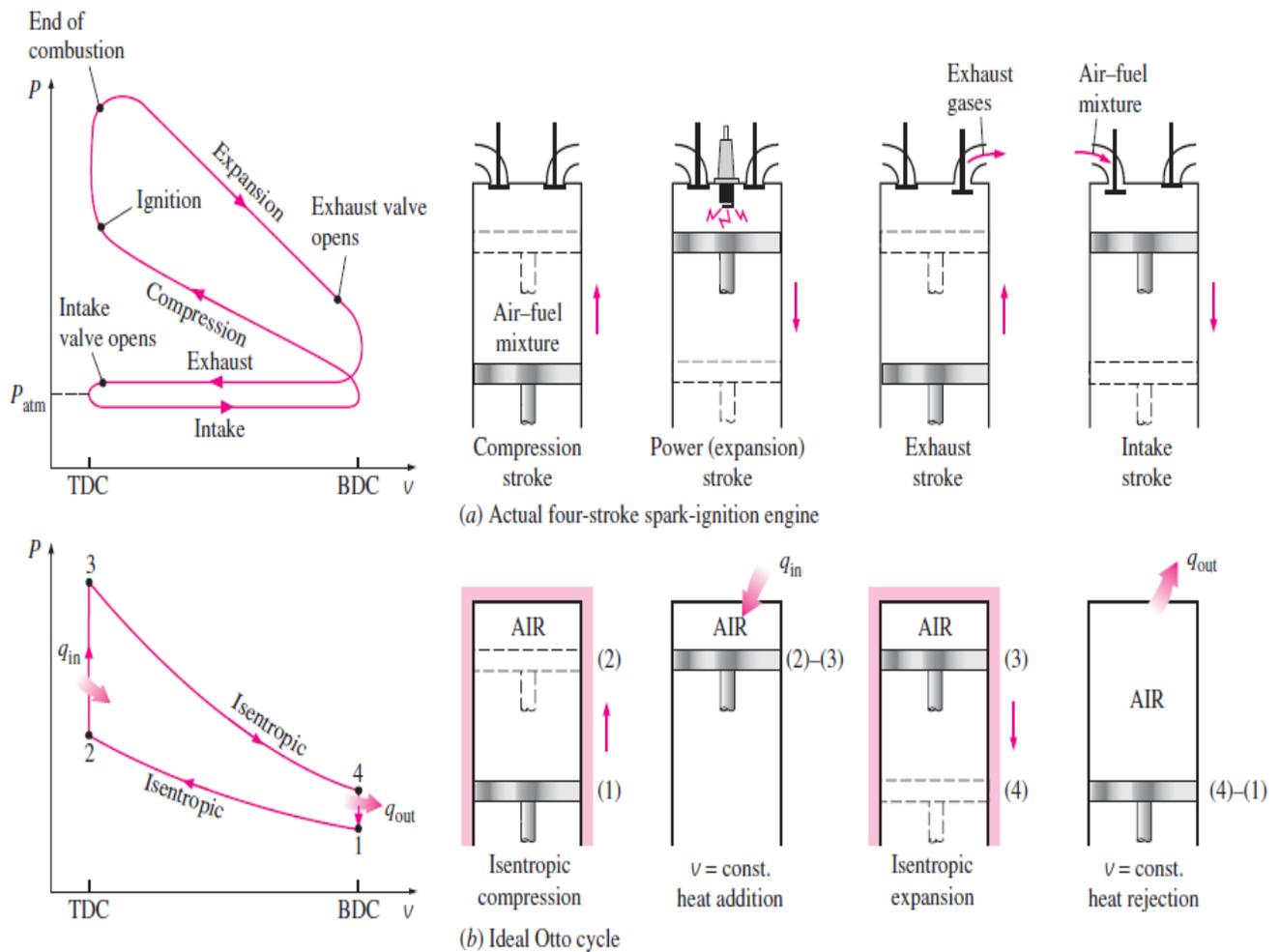


Fig. 6.8 Actual and ideal cycles in spark-ignition engines and their P - v diagrams.

The thermodynamic analysis of the actual four-stroke or two-stroke cycles described is not a simple task. However, the analysis can be simplified

significantly if the air-standard assumptions are utilized. The resulting cycle, which closely resembles the actual operating conditions, is the ideal Otto cycle. It consists of four internally reversible processes:

1-2 Isentropic compression

2-3 Constant-volume heat addition

3-4 Isentropic expansion

4-1 Constant-volume heat rejection

The execution of the Otto cycle in a piston–cylinder device together with a P-v diagram is illustrated in Fig. 6.8. The T-s diagram of the Otto cycle is given in Fig. 6.9. The Otto cycle is executed in a closed system, and disregarding the changes in kinetic and potential energies, the energy balance for any of the processes is expressed, on a unit-mass basis, as

$$(Q_{in} - Q_{out}) + (w_{in} - w_{out}) = \Delta u \text{ kJ/kg}$$

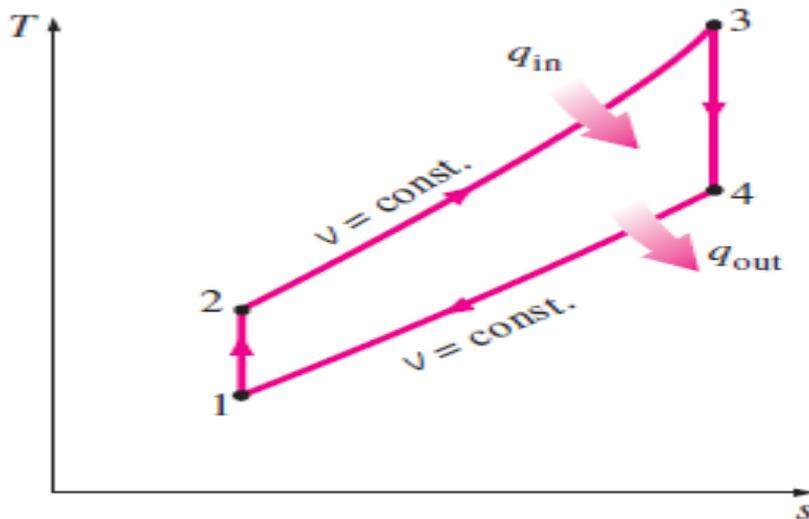


Fig. 6.9 T-s diagram of the ideal Otto cycle.

No work is involved during the two heat transfer processes since both take place at constant volume. Therefore, heat transfer to and from the working fluid can be expressed as

$$Q_{in} = u_3 - u_2 = c_v(T_3 - T_2)$$

$$Q_{out} = u_4 - u_1 = c_v(T_4 - T_1)$$

Then the thermal efficiency of the ideal Otto cycle under the cold air standard assumptions becomes

$$\eta_{Otto} = \frac{w_{net}}{Q_{in}} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{c_v(T_4 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Processes 1-2 and 3-4 are isentropic, and $v_2 = v_3$ and $v_4 = v_1$. Thus,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = \frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma-1} = r_v^{\gamma-1}$$

$$T_1 = T_2 r_v^{\gamma-1}$$

$$T_4 = T_3 r_v^{\gamma-1}$$

$$\therefore (T_3 - T_2) = (T_4 - T_1)/r_v^{\gamma-1}$$

$$\eta_{Otto} = 1 - \left(\frac{1}{r_v^{\gamma-1}}\right)$$

EX. The Ideal Otto Cycle

An ideal Otto cycle has a compression ratio of 8 and maximum volume 676.1 cm³. At the beginning of the compression process, air is at 100 kPa and 17°C, and 800 kJ/kg of heat is transferred to air during the constant-volume heat-addition process. Accounting for the variation of specific heats of air with temperature, determine (a) the maximum temperature and pressure that occur during the cycle, (b) the network output, (c) the thermal efficiency, and (d) the mean effective pressure for the cycle.

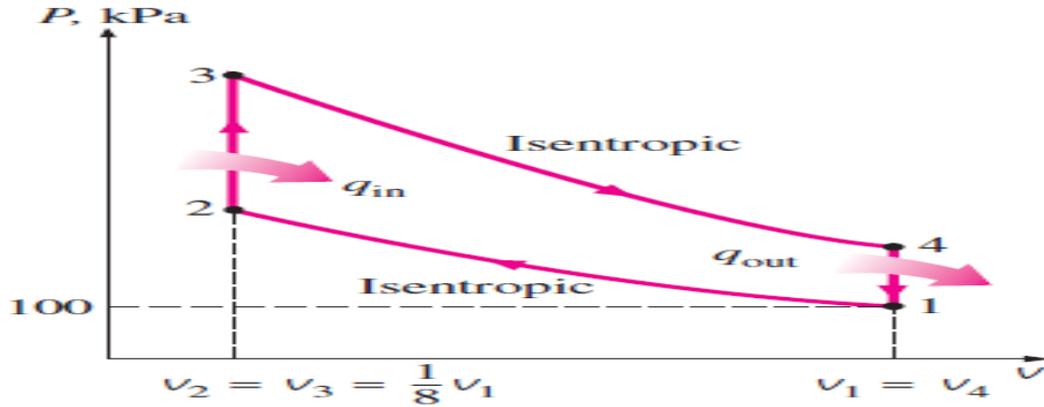


Fig. 6.10 P-v diagram for the Otto cycle discussed in above Ex.

(a) The maximum temperature and pressure in an Otto cycle occur at the end of the constant-volume heat-addition process (state 3). But first we need to determine the temperature and pressure of air at the end of the isentropic compression process (state 2), :Process 1-2 (isentropic compression of an ideal gas):

$$T_1 = 290 \text{ K} \quad u_1 = c_v T_1$$

$$\frac{V_1}{V_2} = r_v = 8$$

$$\frac{T_2}{T_1} = r_v^{\gamma-1}, \therefore T_2 = 652.4 \text{ K}, \quad u_2 = c_v T_2$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}, \quad \therefore P_2 = P_1 \left(\frac{T_2}{T_1} \right) \left(\frac{V_1}{V_2} \right) = 1799.7 \text{ kPa}$$

Process 2-3 constant volume (heat addition)

$$Q_{in} = u_3 - u_2, u_3 = 1275.11 \text{ kJ/kg}, T_3 = 1575.1 \text{ K}$$

$$\frac{P_3 V_3}{T_3} = \frac{P_2 V_2}{T_2}, \quad \therefore P_3 = P_2 \left(\frac{T_3}{T_2} \right) \left(\frac{V_2}{V_3} \right) = 4.345 \text{ kPa}$$

$$\frac{T_4}{T_3} = r_v^{\gamma-1}, \quad T_4 = 795.6 \text{ K}, \quad u_4 = 588.74 \text{ kJ/kg}$$

Process 4-1 (constant-volume heat rejection):

$$-Q_{out} = u_1 - u_4, Q_{out} = u_4 - u_1, Q_{out} = 381.83 \text{ kJ/kg}$$

$$w_{net} = Q_{in} - Q_{out} = 481.17 \text{ kJ/kg}$$

(c) The thermal efficiency of the cycle is determined from its definition:

$$\eta_{otto} = \frac{w_{net}}{Q_{in}} =$$

$$\eta_{otto} = 1 - \left(\frac{1}{r_v^{\gamma-1}} \right) = 56.5\%$$

(d) The mean effective pressure is determined from its definition

$$MEP = \frac{W_{net}}{v_{max} - v_{min}} = \frac{w_{net}}{v_1 \left(1 - \frac{1}{r_v} \right)}$$

$$v_1 = \frac{RT_1}{P_1} = 0.832 \text{ m}^3 / \text{kg}$$

$$MEP = 574 \text{ kpa}$$

6.7 DIESEL CYCLE: THE IDEAL CYCLE FOR COMPRESSION-IGNITION ENGINES.

The Diesel cycle is the ideal cycle for CI reciprocating engines. The CI engine, first proposed by Rudolph Diesel in the 1890s, is very similar to the SI engine discussed in the last section, differing mainly in the method of initiating combustion. In spark-ignition engines (also known as gasoline engines), the air-fuel mixture is compressed to a temperature that is below the auto ignition temperature of the fuel, and the combustion process is initiated by firing a spark plug. In CI engines (also known as diesel engines), the air is compressed to a

temperature that is above the auto ignition temperature of the fuel, and combustion starts on contact as the fuel is injected into this hot air. Therefore, the spark plug and carburetor are replaced by a fuel injector in diesel engines (Fig. 6.11).

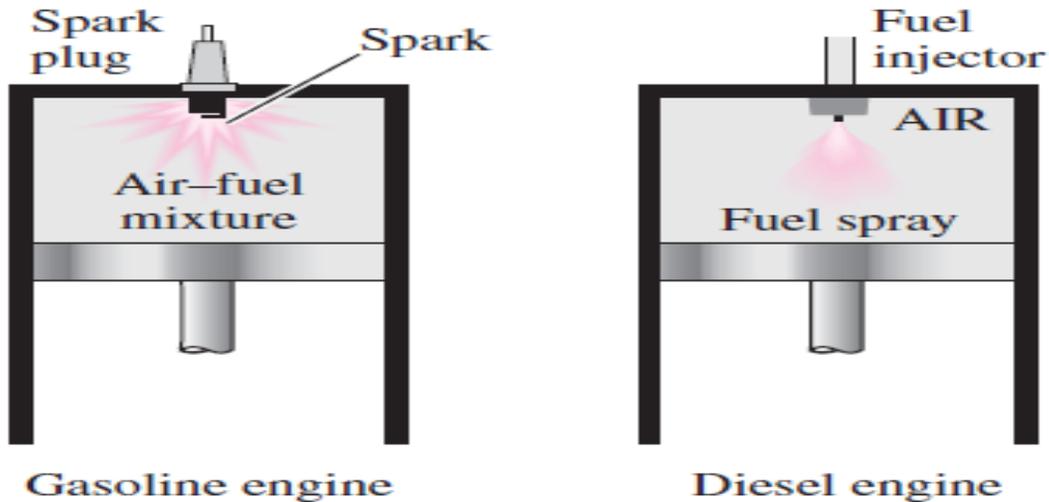


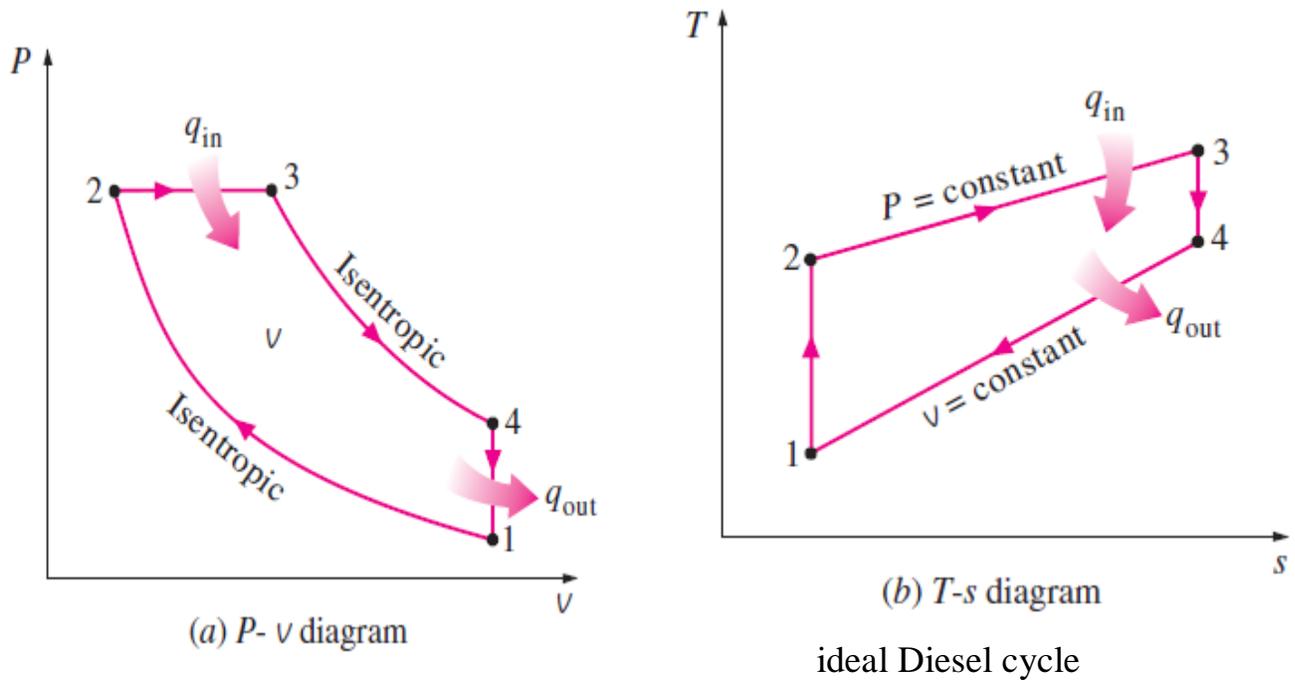
Fig. 6.11 In diesel engine, the spark plug is replaced by a fuel injector, and only air is compressed during the compression process.

In gasoline engines, a mixture of air and fuel is compressed during the compression stroke, and the compression ratios are limited by the onset of auto ignition or engine knock. In diesel engines, only air is compressed during the compression stroke, eliminating the possibility of auto ignition. Therefore, diesel engines can be designed to operate at much higher compression ratios, typically between 12 and 24. Not having to deal with the problem of auto ignition has another benefit: many of the stringent requirements placed on the gasoline can now be removed, and fuels that are less refined (thus less expensive) can be used in diesel engines.

The fuel injection process in diesel engines starts when the piston approaches TDC and continues during the first part of the power stroke. Therefore, the combustion process in these engines takes place over a longer interval. Because of this longer duration, the combustion process in the ideal Diesel cycle is approximated as a constant-pressure heat-addition process. In fact, this is the only process where the Otto and the Diesel cycles differ. The remaining three processes are the same for both ideal cycles. That is, process 1-2 is isentropic compression, 3-4 is isentropic

expansion, and 4-1 is constant-volume heat rejection. The similarity between the two cycles is also apparent from the P-v and T-s diagrams of the Diesel cycle, shown in Fig. 6.12.

Fig. 6.12 T-s and P-v diagrams for the



Noting that the Diesel cycle is executed in a piston–cylinder device, which forms a closed system, the amount of heat transferred to the working fluid at constant pressure and rejected from it at constant volume can be expressed as

$$(Q_{in} - w_{b,out}) = (u_3 - u_2), \quad \therefore Q_{in} = P_2(V_3 - V_2) + (u_3 - u_2)$$

$$Q_{in} = h_3 - h_2 = c_p(T_3 - T_2)$$

And

$$Q_{out} = u_4 - u_1 = c_v(T_4 - T_1)$$

Then the thermal efficiency of the ideal Diesel cycle under the cold-air standard

assumptions becomes

$$\begin{aligned}\eta_{Diesel} &= \frac{w_{net}}{Q_{in}} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{\gamma(T_3 - T_2)} \\ &= 1 - \frac{T_1(T_4/T_1 - 1)}{\gamma T_2(T_3/T_2 - 1)}\end{aligned}$$

We now define a new quantity, the cutoff ratio β , as the ratio of the cylinder volumes after and before the combustion process:

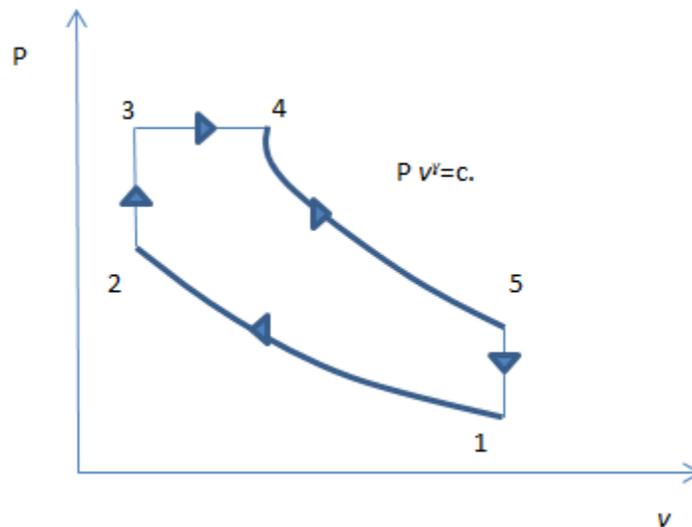
$$\beta = \frac{v_3}{v_2}$$

$$\eta_{Diesel} = 1 - \frac{1}{r_v^{\gamma-1}} \left[\frac{\beta^\gamma - 1}{\gamma(\beta - 1)} \right] =$$

$$\eta_{Diesel} < \eta_{Otto}$$

The dual combustion cycle

Modern oil engines although still called diesel engines are more closely derived from an engine invented by Ackroyd-Stuart in 1888.



process 1-2 is isentropic compression,

process 2-3 is reversible constant volume heating

process 3-4 is reversible constant pressure heating

process 4-5 is isentropic expansion.

process 5-1 is reversible constant-volume cooling (heat rejection).

The compression ratio r_v of the engine:

$$r_v = \frac{v_{max}}{v_{min}} = \frac{v_1}{v_2}$$

The pressures ratio k of the engine:

$$k = \frac{p_3}{p_2}$$

the cutoff ratio (volumes ratio) β

$$\beta = \frac{v_4}{v_3}$$

$$\therefore \eta_{Dual} = 1 - \frac{k\beta^\gamma - 1}{[(k - 1) + \gamma k(\beta - 1)]r_v^{\gamma-1}}$$

The heat supplied

$$Q_1 = c_v(T_3 - T_2) + c_p(T_4 - T_3)$$

The heat rejected

$$Q_2 = c_v(T_5 - T_1)$$

$$i. e k = \frac{p_3}{p_2} = 1, \text{ when } p_3 = p_2$$

The Stirling and Ericsson cycles

The cycles which have efficiency equal to that of the Carnot cycle have been defined and are known as the Stirling and Ericsson cycles and they are superior to the Carnot cycle in that they have higher work ratios.

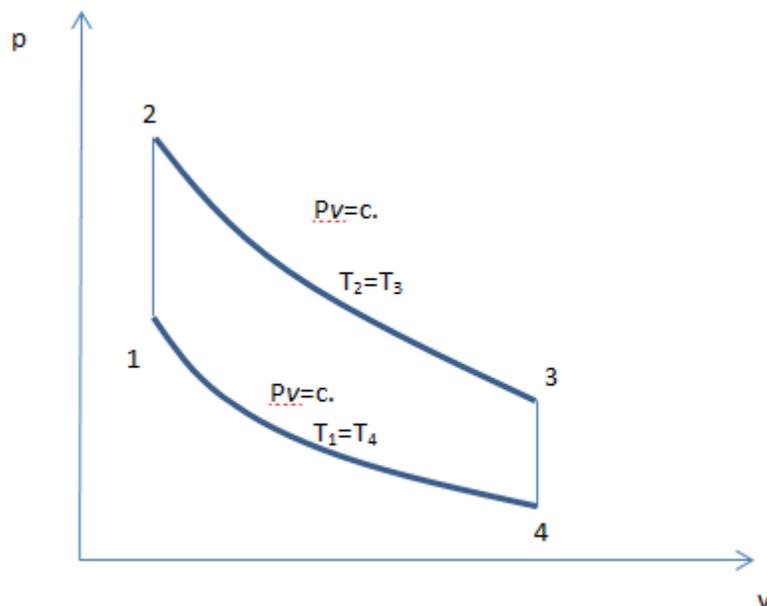
Heat is supplied to the working fluid, usually hydrogen or helium, from an external source, process 2-3 as the gas expands isothermally ($T_2=T_3$), and heat is rejected to an external sink, process 4-1 as the gas is compressed isothermally ($T_1=T_4$). The two isothermals are connected by the reversible constant volume process 1-2, 3-4 during which the temperature changes are equal to (T_2-T_1).

The heat rejected during 3-4, is used heat the gas during 1-2

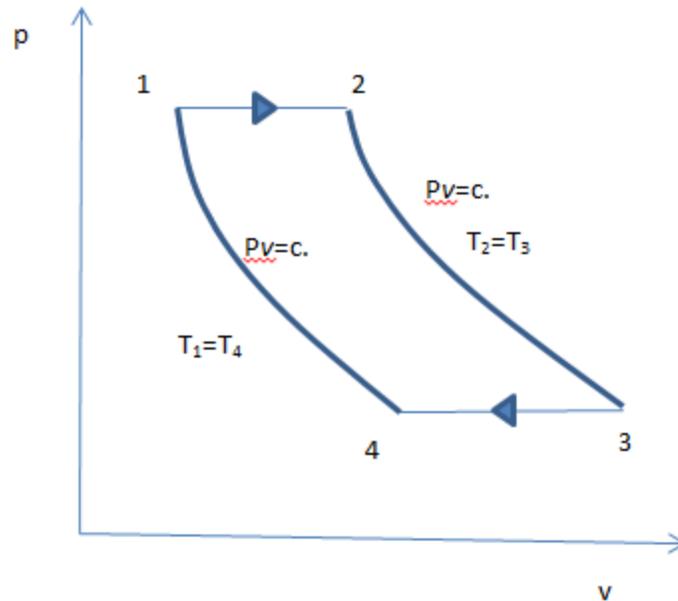
$$Q_{2,34} = c_v(T_2 - T_1)$$

The heat supplied 1-2

$$Q_{1,12} = c_v(T_2 - T_1) = Q_{2,34}$$



The similar Ericson cycle is similar to the Stirling cycle except that the two isothermal process are connected by constant pressure processes.



Heat supplied from the heat source,

$$Q_{2-3} = W_{2-3} = RT_2 \ln \frac{p_2}{p_3}$$

Similarly, the heat rejected,

$$Q_{4-1} = W_{4-1} = RT_1 \ln \frac{p_1}{p_4}$$

For the complete cycle,

Net work done = Net heat supplied

$$W = Q_{2-3} - Q_{4-1}$$

The cycle efficiency,

$$\eta = \frac{W}{Q_{2-3}}$$

$$\eta = \frac{Q_{2-3} - Q_{4-1}}{Q_{2-3}}$$

$$\eta = 1 - \frac{Q_{4-1}}{Q_{2-3}}$$

$$\eta = 1 - \frac{R T_1 \ln \frac{p_1}{p_4}}{R T_2 \ln \frac{p_2}{p_3}}$$

For the constant volume process 1-2,

$$\frac{p_2}{p_1} = \frac{T_2}{T_1} \text{ and for process 3-4, } \frac{p_3}{p_4} = \frac{T_3}{T_4} = \frac{T_2}{T_1}$$

$$\frac{p_2}{p_1} = \frac{p_3}{p_4} \text{ and } \frac{p_2}{p_3} = \frac{p_1}{p_4}$$

$$\therefore \eta = 1 - \frac{T_1}{T_2} = \text{Carnot cycle}$$

$$\text{The work ratio} = \frac{W_{2-3} - W_{4-1}}{W_{2-3}}$$

$$\text{The work ratio} = 1 - \frac{W_{4-1}}{W_{2-3}} = 1 - \frac{Q_{4-1}}{Q_{2-3}} = 1 - \frac{T_1}{T_2}$$