Acta Physica Hungarica 72 (2-4), pp. 123-132 (1992)

THE CORRECTION TERM IN A DISLOCATION CONTAINING LATTICE

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(Received 23 January 1992)

The correction term of a sample having dislocation (strain field as well as core) has been studied at low temperatures on the basis of the Callaway integral as well as in the frame of the generalized Callaway integral. Assuming four types of the scattering mechanisms, viz. boundary, dislocation, point defect and three phonon, analytical expressions for the correction term are given under two different conditions. The expressions derived give results in agreement with the findings of previous works.

1. Introduction

Callaway [1] was the first who distinguished the three phonon normal processes from the three umklapp processes. He derived an expression for the lattice thermal conductivity of an insulator as a sum of two parts. The first one is attributed to the combined scattering relaxation rates, whereas the second part is very complicated known as the correction term (ΔK) due to the three phonon normal processes.

Kosarev et al [2] and Parrot [3] have published a generalization of Callaway's approach to thermal conductivity when different polarizations are taken into account. Later on, the generalized expression was modified by Dubey [4] introducing the dispersion of phonons.

The lattice thermal conductivity due to the correction term has been studied in the frame of the Callaway integral as well as in the frame of the generalized Callaway integral [5–11], and it is usually found that the contribution to the lattice thermal conductivity arising from the correction term is negligibly small. (Exceptions are solid He [9], LiF [10] and solid HD [11]). These studies are confined to samples of perfect structures only. In none of them was an analytical expression obtained for the correction term in a lattice having dislocations (strain field and core). Recently, the present author [12,13] studied the contribution of the correction term for a sample containing core dislocation in the frame of the Callaway integral as well as the generalized Callaway integral. It was found that analytical expressions are very necessary to avoid the complicated integrals of the correction term.

The aim of the present investigation is to obtain a simple analytical expression for the correction term on the basis of the Callaway integral and also the generalized Callaway integral at very low temperatures.

2. Analytical expression for ΔK in the frame of the Callaway integral

According to Callaway [1], the lattice thermal conductivity of an insulator can be written as

$$K = c_0 L_1 + \Delta K, \tag{1}$$

where

$$\Delta K = c_0 L_2^2 / L_3, \tag{2}$$

$$L_1 = \int_0^{\Theta_D/T} (\tau_N^{-1} + \tau_R^{-1})^{-1} F(x) dx, \tag{3}$$

$$L_2 = \int_0^{\Theta_D/T} \tau_N^{-1} (\tau_N^{-1} + \tau_R^{-1})^{-1} F(x) dx, \tag{4}$$

$$L_3 = \int_0^{\Theta_D/T} \tau_R^{-1} \tau_N^{-1} (\tau_N^{-1} + \tau_R^{-1})^{-1} F(x) dx. \tag{5}$$

$c_0 = (K_B / 2\pi^2 v)(K_B T/\hbar)^3$, $F(x) = x^4 e^x (e^x - 1)^{-2}$, $x = \hbar\omega / K_B T$, ω is the phonon frequency, Θ_D is the Debye temperature of the sample, v is the velocity of the phonon, τ_N^{-1} is the three phonon normal processes scattering relaxation rate and τ_R^{-1} is the total scattering relaxation rate due to all momentum nonconserving processes. The expressions used for the scattering relaxation rate can be derived as

$$\tau_R^{-1} = \tau_B^{-1} + \tau_{pt}^{-1} + \tau_{ds}^{-1} + \tau_{dc}^{-1} + \tau_U^{-1}, \tag{6}$$

$$\tau_{pt}^{-1} = A\omega^4 = Dx^4, \tag{7}$$

$$\tau_{ds}^{-1} = a'\omega = a_1 x, \tag{8}$$

$$\tau_{dc}^{-1} = d\omega^3 = cx^3, \tag{9}$$

$$\tau_N^{-1} = B\omega^2 T^3 = b_1 x^2, \tag{10}$$

$$\tau_U^{-1} = B'\omega^2 T^3 = b_2 x^2, \tag{11}$$

$$E = b_1 + b_2, \tag{12}$$

where τ_B^{-1}, τ_{pt}^{-1}, τ_{ds}^{-1}, τ_{dc}^{-1} and τ_U^{-1} are the scattering relaxation rates ascribed to boundary [13], point defect [15], strain field dislocation [15], core dislocation [15] and three phonon umklapp processes [1], respectively. A, a', d, B and B' are scattering strengths of the respective processes.

The analytical expressions have been obtained in the low temperature approximation under two different conditions, i.e. $\tau_{ds}^{-1} \gg (\tau_B^{-1} + \tau_{dc}^{-1})$ and $\tau_B^{-1} \gg (\tau_{dc}^{-1} + \tau_{ds}^{-1})$.

(A) $\tau_{ds}^{-1} \gg (\tau_B^{-1} + \tau_{dc}^{-1})$

For this case, the simplified forms of L_2 and L_3 can be written as

$$L_2 = \frac{b_1 I_5}{a_1}\left[1 - G_1 F_5^4 - G_3 F_5^8 - G_2 F_5^6 - G_4 F_5^7\right], \tag{13}$$

$$\begin{aligned} L_3 = b_1 I_6 \,[\, 1 &- \frac{P G_2}{1+P} F_6^7 - G_4^2 F_6^{10} - 2G_1 G_4 F_6^7 - 2G_3 G_4 F_6^{11} - \frac{(2+P)}{(1+P)} G_2 G_4 F_6^9 \\ &- G_1^2 F_6^4 - 2G_1 G_3 F_6^8 - G_1 G_2 \frac{(2+P)}{(1+P)} - G_3^2 F_6^{12} - \frac{(2+p)}{(1+P)} G_2 G_3 F_6^{10} \\ &- \frac{G_2^2}{(1+P)} F_6^8 \,], \end{aligned} \tag{14}$$

where $G_1 = \tau_B^{-1}/a_1$, $G_2 = E/a_1$, $G_3 = D/a_1$, $G_4 = c/a_1$, $P = b_1/b_2$, $F_n^m = I_m/I_n$, m and $n = 1, 2, 3, \ldots$, etc. and the I's are integrals which can be expressed as

$$I_r = \int_0^{\Theta_D/T} x^r e^x (e^x - 1)^{-2} dx,$$

where $r = 2, 3, 4, \ldots$, etc. At very low temperatures, the upper limit of the integrals can be taken as infinity. Thus I_r can be evaluated as

$$I_r = \int_0^{\infty} x^r e^x (e^x - 1)^{-2} dx = r! \sum_{j=1}^{\infty} \frac{1}{j^r}. \tag{15}$$

It is observed that the contribution of the three phonon umklapp processes scattering relaxation rate is very small compared to τ_R^{-1} and it can be ignored. A further approximation can be made by neglecting the lower order term and then the expression for ΔK is given by

$$\Delta K = \frac{c_0 b_1 I_5 F_6^5}{a_1^2}\left[1 - 2G_1 F_5^4 - 2G_3 F_5^8 - 2G_2 F_5^6 + G_2 F_6^7 - 2G_4 F_5^7\right], \tag{16}$$

(B) $\tau_B^{-1} \gg (\tau_{dc}^{-1} + \tau_{ds}^{-1})$

Applying this condition and evaluating (4) and (5), we get

$$L_2 = b_1 \tau_B I_6 \left[1 - c\tau_B F_6^9 - a_1 \tau_B F_6^7 - D\tau_B F_6^{10} - E\tau_B F_6^8\right], \tag{17}$$

$$\begin{aligned} L_3 = b_1 I_6 \,[1 &- b_1 \tau_B F_6^8 - c^2 \tau_B^2 F_6^{12} - 2a_1 c \tau_B^2 F_6^{10} - 2Dc\tau_B^2 F_6^{12} - a_1^2 \tau_B^2 F_6^8 \\ &- (E + b_2) c \tau_B^2 F_6^{11} - 2a_1 D \tau_B^2 F_6^{11} - (E + b_2) a_1 \tau_B^2 F_6^9 - D^2 \tau_B^2 F_6^{14} \\ &- (E + b_2) D \tau_B^2 F_6^{12} - E b_2 \tau_B^2 F_6^{10}]. \end{aligned} \tag{18}$$

Using these equations, ΔK is expressed as

$$\Delta K = c_0 b_1 \tau_B^2 I_6 \left[1 - 2c\tau_B F_6^9 - 2a_1\tau_B F_6^7 - 2D\tau_B F_6^{10} - b_1\tau_B F_6^8\right]. \quad (19)$$

With the help of Eq. (15), one can get an expression for ΔK as follows:

$$\Delta K = 720 c_0 b_1 \tau^2 \left[1 - 1008c\tau_B - 14a_1\tau_B - 10080D\tau_B + 56b_1\tau_B\right]. \quad (20)$$

In the absence of strain field dislocation, ΔK reduces to

$$\Delta K = c_0 b_1 \tau_B^2 I_6 \left[1 - 2c\tau_B F_6^9 - 2D\tau_B F_6^{10} - b_1\tau_B F_6^8\right], \quad (21)$$

which is similar to that obtained by Awad [12] for a sample having core dislocation in the frame of the Callaway integral.

The expression for ΔK will be

$$\Delta K = c_0 b_1 \tau_B^2 I_6 \left[1 - 2D\tau_B^2 F_6^{10} - b_1\tau_B^2 F_6^8\right] \quad (22)$$

for a sample without dislocation, which is the same as that obtained by Dubey [8].

3. Analytical expression for ΔK in the frame of the generalized Callaway integral

The generalized form of the Callaway integral of the lattice thermal conductivity can be given as

$$K = c_1 \left[\sum_s (1/v_s) \int_0^{\Theta_s/T} (\tau_{N,s}^{-1} + \tau_{R,s}^{-1})^{-1} (1 + R_s x^2)^2 (1 + 3R_s x^2)^{-1} F(x) dx + \Delta K\right], \quad (23)$$

where $c_1 = K_B/6\pi^2 v_s)(K_B T/\hbar)^3$, R is a constant depending on the dispersion curve of the sample, suffix s represents the mode of phonon and all other terms have their meaning as in Section 2.

According to Dubey [4], ΔK can be written as

$$\Delta K = c_1 \left[2(T_1 + T_2) + (L_1 + L_2)\right]^2 / \left[2(T_3 + T_4) + (L_3 + L_4)\right]. \quad (24)$$

It was found that the contributions of T_2, T_4, L_2 and L_4 are very small in comparison to the contributions of T_1, T_3, L_1 and L_3 due to their integration limit. Thus ΔK becomes:

$$\Delta K = c_1 \left[(2T_1 + L_1)^2/(2T_3 + L_3)\right], \quad (25)$$

where

$$T_1 = (1/v_{T_1}^3) \int_0^{\Theta_1/T} \tau_{N,T}^{-1}(\tau_{N,T}^{-1} + \tau_{R,T}^{-1})^{-1}(1 + R_1x^2)^3 F(x)dx, \tag{26}$$

$$T_3 = (1/v_{T_1}^5) \int_0^{\Theta_1/T} \tau_{N,T}^{-1}\tau_{R,T}^{-1}(\tau_{N,T}^{-1} + \tau_{R,T}^{-1})^{-1}(1 + R_1x^2)^4(1 + 3R_1x^2)^4 F(x)dx, \tag{27}$$

$$L_1 = (1/v_{L_1}^3) \int_0^{\Theta_3/T} \tau_{N,L}^{-1}(\tau_{N,L}^{-1} + \tau_{R,L}^{-1})^{-1}(1 + R_3x^2)^3 F(x)dx, \tag{28}$$

$$L_3 = (1/v_{L_1}^5) \int_0^{\Theta_3/T} \tau_{N,L}^{-1}\tau_{R,L}^{-1}(\tau_{N,L}^{-1} + \tau_{R,L}^{-1})^{-1}(1 + R_3x^2)^4(1 + 3R_3x^2)^4 F(x)dx. \tag{29}$$

Θ's are the characteristic temperatures, suffixes T and L represent transverse and longitudinal phonons, respectively. The expressions used for the above stated scattering relaxation rate can be expressed as

$$\tau_{N,L}^{-1} = B_1\omega^2T^3 = b_3x^2, \tag{30}$$

$$\tau_{U,L}^{-1} = B_2\omega^2T^3e^{-\Theta D/\alpha T} = b_4x^2, \tag{31}$$

$$\tau_{N,T}^{-1} = B_3\omega T^4 = b_5x, \tag{32}$$

$$\tau_{U,T}^{-1} = B_4\omega T^4e^{-\Theta_D/\alpha T} = b_6x, \tag{33}$$

where B_1 and B_2 are the three phonon normal and umklapp processes scattering strengths, respectively, for longitudinal phonons, B_3 and B_4 are the same for the transverse phonons, α is a constant [15] and other terms have been defined in Section 2.

As stated earlier, our interests are confined to low temperature only, therefore our study is limited to two approximations.

(A) $\tau_B^{-1} \gg (\tau_{cd}^{-1} + \tau_{sd}^{-1})$

In the framework of the above approximation, the integrals T_1, L_1, T_3 and L_3 can be given

$$T_1 = \frac{b_5\tau_B I_5}{v_{T_1}^3}\left[X_5^7(R) - H_1F_5^8X_8^{10}(R_1) - H_2F_5^9X_9^{11}(R_1) - (H_3 + H_5)F_5^6X_6^8(R_1)\right], \tag{34}$$

$$L_1 = \frac{b_3\tau_B I_6}{v_{L_1}^3}\left[X_6^8(R_3) - H_1F_6^9X_9^{11}(R_3) - H_2F_6^{10}X_{10}^{12}(R_3) - H_4F_6^8X_8^{10}(R_3) - H_5F_6^7X_7^9(R_3)\right], \tag{35}$$

$$T_3 = \frac{b_5 I_5}{v_{T_1}^5}\Big[Y_5^7(R_1) - \frac{P'H_3}{(1+p')}F_5^6Y_6^8(R_1) - H_1^2F_5^{11}Y_{11}^{13}(R_1) - 2H_1H_5F_5^9Y_9^{11}(R_1)$$
$$- 2H_1H_2F_5^{12}Y_{12}^{14}(R_1) - \frac{(2+P')}{(1+P')}H_1H_3F_5^9Y_9^{11}(R_1) - H_5^2F_5^7Y_7^9(R_1)$$
$$- 2H_2H_5F_5^{10}Y_{10}^{12}(R_1) - \frac{(2+P')}{(1+P')}H_3H_5F_5^7Y_7^9(R_1) - H_2^2F_5^{13}Y_{13}^{15}(R_1)$$
$$-\frac{(2+P')}{(1+P')}H_2H_3F_5^{10}Y_{10}^{12}(R_1) - \frac{H_3^2}{(1+P')}F_5^7Y_7^9(R_1)\Big], \tag{36}$$

$$L_3 = \frac{b_3 I_6}{v_{L_1}^5}\Big[Y_6^8(R_3) - \frac{P_1}{(1+P_1)}H_4F_6^8Y_8^{10}(R_3) - H_1^2F_6^{12}Y_{12}^{14}(R_3)$$
$$- 2H_1H_5F_6^{10}Y_{10}^{12}(R_3) - 2H_1H_2F_6^{13}Y_{13}^{15}(R_3) - \frac{(2+P_1)}{(1+P_1)}H_1H_4F_6^{11}Y_{11}^{13}(R_3)$$
$$- H_5^2F_6^8Y_8^{10}(R_3) - H_5^2F_6^8Y_8^{10}(R_3) - H_2H_5F_6^{11}Y_{11}^{13}(R_3)$$
$$- \frac{(2+P_1)}{(1+P_1)}H_4H_5F_6^9Y_9^{11}(R_3) - H_2^2F_6^{14}Y_{14}^{16}(R_3) - \frac{(2+P_1)}{(1+P_1)}H_2H_4F_6^{12}Y_{12}^{14}(R_3)$$
$$-\frac{H_4^2}{(1+P_1)}F_6^{10}Y_{10}^{12}(R_3)\Big], \tag{37}$$

where

$$X_m^n(R_i) = 1 + 3R_iF_m^n + 3R_i^2F_m^{n+2} + R_i^3F_m^{n+4},$$
$$Y_m^n = 1 + 7R_iF_m^n + 18R_i^2F_m^{n+2} + 22R_i^3F_m^{n+4} + 7R_i^4F_m^{n+5} + 3R_i^5F_m^{n+8},$$
$$H_1 = c\tau_B, \quad H_2 = D\tau_B, \quad H_3 = (b_5+b_6)\tau_B, \quad H_4 = b_3\tau_B,$$
$$H_5 = a_1\tau_B, \quad P' = \frac{b_5}{b_6} \quad \text{and} \quad P_1 = \frac{b_3}{b_4}.$$

At low temperatures, the contribution of the three phonon umklapp process scattering relaxation rate is very small to the combined scattering relaxation rate and it can be ignored in the calculation of ΔK. Neglecting τ_U^{-1} as well as other lower terms, ΔK can be approximated to

$$\Delta K = 3c_1b_5\tau_B^2(Z_1 + Z_2)/(1 + a/2)v_sZ_3, \tag{38}$$

where

$$Z_1 = N_1 \Big[N_1 - 2H_1 F_5^8 (1 + \frac{qa^3}{2} F_8^9) - 2H_5 F_5^6 (1 + \frac{qa^3}{2} F_6^7) - 2H_2 F_5^9 (1 + \frac{qa^3}{2} F_9^{10})$$
$$-2b_5 \tau_B F_5^6 (1 + \frac{q^2 a^3}{2} F_6^8) \Big] , \quad (39)$$

$$Z_2 = 6R_1 F_5^7 \Big[(1 + \frac{qa^3}{2} F_5^6 + \frac{qa^3}{2} c' F_7^8 + \frac{q^2 a^6}{2} c' F_7^8 F_5^6) - H_1 F_7^{10} \{ 1 + F_{10}^7 F_5^8$$
$$+ \frac{qa^3}{2} F_5^6 (1 + F_{10}^7 F_6^9) + \frac{qa^3}{2} c' F_{10}^{11} (1 + F_5^8 F_{11}^8) + \frac{q^2 a^6}{4} c' F_5^6 F_{10}^{11} (1 + F_6^8 F_{11}^9) \Big\}$$
$$- H_5 F_7^8 \Big\{ 1 + F_8^7 F_5^6 + \frac{qa^3}{2} F_5^6 (1 + F_8^7 F_6^7) + \frac{qa^3}{2} c' F_8^9 (1 + F_5^6 F_9^8)$$
$$+ \frac{q^2 a^6}{4} c' F_5^6 F_8^9 (1 + F_6^7 F_9^8) \Big\} - H_2 F_7^{11} \Big\{ 1 + F_{11}^7 F_5^9 + \frac{qa^3}{2} F_5^6$$
$$(1 + F_{11}^7 F_6^{10}) + \frac{qa^3}{2} c' F_{11}^{12} (1 + F_5^9 F_{12}^8) + \frac{q^2 a^6}{4} c' F_5^6 F_{11}^{12} (1 + F_6^8 F_{12}^{10}) \Big\} - b_1 \tau_B F_7^8$$
$$\{ 1 + F_8^7 F_5^6 + \frac{qa^3}{2} F_5^6 (1 + c') + \frac{q^2 a^3}{2} F_5^7 (1 + c' F_7^5 F_8^{10})$$
$$+ \frac{q^2 a^6}{4} c' F_5^8 (1 + F_8^6 F_8^{10}) \Big\} \Big] , \quad (40)$$

$$Z_3 = N_2 - b_5 \tau_B F_5^6 (1 + \frac{q^2 a^5}{2} F_6^8 + 7R_1 F_5^7 \Big\{ (1 + \frac{qa^5}{2} c' F_7^8) - b_5 \tau_B F_7^8$$
$$(1 + \frac{q^2 a^5}{2} c' F_8^{10}) \Big\} . \quad (41)$$

$N_1 = 1 + \frac{qa^3}{2} F_5^6$, $N_2 = 1 + \frac{qa^5}{2} F_5^6$, $c' = \frac{R_1}{R_3}$, $a = \frac{v_{T_1}}{v_{L_1}}$ and $q = \frac{b_3}{b_5}$. With the help of Eq. (15) and neglecting the dispersion of phonons, Eq. (38) reduces to

$$\Delta K = 960 c_1 b_5 v_s^{-1} \tau_B^2 [1 - 924 c \tau_B - 13.3 a_1 \tau_B - 9072 D \tau_B - 130.5 b_5 \tau_B]. \quad (42)$$

In the absence of the strain field dislocation, we get

$$\Delta K = 960 c_1 b_5 v_s^{-1} \tau_B^2 [1 - 924 c \tau_B - 9072 D \tau_B - 130.5 b_5 \tau_B]. \quad (43)$$

A similar expression was obtained by Awad [13] for a crystal having core dislocations. The expression for ΔK reduces to

$$\Delta K = 960 c_1 b_5 v_s^{-1} \tau_B^2 [1 - 9072 D \tau_B - 130.5 b_5 \tau_B] \quad (44)$$

for a sample having perfect structure, which is similar as the one obtained by Dubey [11].

(B) $\tau_{sd}^{-1} \gg (\tau_B^{-1} + \tau_{cd}^{-1})$

The terms T_1, L_1, T_3 and L_3 can also be approximated as

$$T_1 = \frac{b_5 I_4}{a_1 v_{T_1}^3}\left[X_4^6(R_1) - G_1 F_4^3 X_3^5(R_1) - G_6 X_4^6(R_1) - G_3 F_4^7 X_7^9(R_1) - G_4 F_4^6 X_6^8(R_1)\right], \quad (45)$$

$$L_1 = \frac{b_3 I_5}{a_1 v_{L_1}^3}\left[X_5^7(R_3) - G_1 F_5^4 X_4^6(R_3) - G_3 F_5^8 X_8^{10}(R_3) - G_4 F_5^7 X_7^9(R_3) - G_5 F_5^6 X_6^8(R_3)\right], \quad (46)$$

$$\begin{aligned} T_3 = \frac{b_5 I_5}{v_{T_1}^5}\Big[&(1 - \frac{P'}{(1+P')} G_6 Y_5^7(R_1) - G_1^2 F_5^3 Y_3^5(R_1) - 2G_1 G_4 F_5^6 Y_6^8(R_1) \\ &- 2G_1 G_3 F_5^7 Y_7^9(R_1) - \frac{(2+P')}{(1+P')} G_1 G_6 F_5^4 Y_4^6(R_1) - G_4^2 F_5^9 Y_9^{11}(R_1) \\ &- 2G_3 G_4 F_5^{10} Y_{10}^{12}(R_1) - \frac{(2+P')}{(1+P')} G_6 G_4 F_5^7 Y_7^9(R_1) - G_3^2 F_5^{11} Y_{11}^{13}(R_1) \\ &- \frac{(2+P')}{(1+P')} G_6 G_3 F_5^8 Y_8^{10}(R_1) - \frac{G_6^2}{(1+P')} Y_5^7(R_1)\Big], \end{aligned} \quad (47)$$

$$\begin{aligned} L_3 = \frac{b_3 I_6}{v_{L_1}^5}\Big[&Y_6^8(R_3) - \frac{P_1}{(1+P_1)} G_5 F_6^7 Y_7^9(R_3) - G_1^2 F_6^4 Y_4^6(R_3) - 2G_1 G_4 F_6^7 Y_7^9(R_3) \\ &- 2G_1 G_3 F_6^8 Y_8^{10}(R_3) - \frac{(2+P_1)}{(1+P_1)} G_1 G_5 Y_6^8(R_3) - G_4^2 F_6^{10} Y_{10}^{12}(R_3) \\ &- 2G_3 G_4 F_6^{11} Y_{11}^{13}(R_3) - \frac{(2+P_1)}{(1+P_1)} G_4 G_5 F_6^9 Y_9^{11}(R_3) - G_3^2 F_6^{12} Y_{12}^{14}(R_3) \\ &- \frac{(2+P_1)}{(1+P_1)} G_3 G_5 F_6^{10} Y_{10}^{12}(R_3) - \frac{G_5^2}{(1+P_1)} F_6^8 Y_8^{10}(R_3)\Big], \end{aligned} \quad (48)$$

where $G_5 = (b_3 + b_4)/a_1$ and $G_6 = (b_5 + b_6)/a_1$. Neglecting the lower value terms, the expression for ΔK can be simplified as

$$\Delta K = \left[\frac{3c_1 b_5 I_4 F_5^4}{a_1^2 v_s (1 + a/2)}\right]\left[\frac{S_1 + S_2}{S_3}\right], \quad (49)$$

where

$$S_1 = N_3\Big[N_3 - 2G_1 F_4^3(1 + \frac{qa^3}{2} F_3^4) - 2G_3 F_4^7(1 + \frac{qa^3}{2} F_7^8) - 2G_4 F_4^6(1 - \frac{qa^3}{2} F_6^7)$$

$$-2G_7(1+\frac{q^2a^3}{2}F_4^6)\Big], \tag{50}$$

$$S_2 = 6R_1F_4^6\Big[1+\frac{qa^3}{2}F_4^5+\frac{qa^3}{2}c'F_6^7+\frac{q^2a^6}{4}c'F_4^5F_6^7 - G_1F_6^5\{1+F_5^6F_4^3$$
$$+\frac{qa^3}{2}F_4^5(1+F_5^6F_5^4)+\frac{qa^3}{2}c'F_5^6(1+F_4^3F_6^7)+\frac{q^2a^6}{4}c'F_4^6(1+F_4^3F_6^7)\Big\}$$
$$-G_3F_6^9\{1+F_9^6F_4^7+\frac{qa^3}{2}F_4^5(1+F_9^6F_5^8)+\frac{qa^3}{2}c'F_9^{10}(1+F_4^7F_{10}^7)$$
$$+\frac{q^2a^6}{4}c'F_4^5F_9^{10}(1+F_5^8F_{10}^7)\Big\} - G_4F_6^8\Big\{1+F_8^6F_4^6+\frac{qa^3}{2}F_4^5(1+F_8^6F_5^7)$$
$$+\frac{qa^3}{2}c'F_8^9(1+F_4^6F_9^7)+\frac{q^2a^6}{4}c'\,F_4^5F_8^9(1+F_5^7F_9^7)\} - G_7\Big\{2+\frac{qa^3}{2}F_4^5$$
$$\cdot(1+c'F_5^4F_6^7)+\frac{q^2a^3}{2}F_4^6(1+c'F_6^4F_6^8)+\frac{q^2a^6}{4}c'F_4^7(1+F_7^5F_6^8)\Big\}\Big], \tag{51}$$

$$S_3 = N_2 - G_6(1+\frac{q^2a^5}{2}F_5^7)+7R_1F_5^7\Big\{1+\frac{qa^5}{2}c'F_7^8 - G_7(1+\frac{q^2a^5}{2}c'F_7^9)\Big\}, \tag{52}$$

$$N_3 = 1+\frac{qa^3}{2}F_4^5 \quad \text{and} \quad G_7 = b_5/a_1.$$

Neglecting the dispersion and with some mathematical manipulation, Eq. (49) can be expressed as

$$\Delta K = 31.86c_1b_5v_s^{-1}a_1^{-2}\left[1-0.45\tau_B^{-1}/a_1 - 76.7c/a_1 - 572.5D/a_1 - 3.42b_5/a_1\right]. \tag{53}$$

In the absence of core dislocations, ΔK will be:

$$\Delta K = 31.86c_1b_5v_s^{-1}a_1^{-2}\left[1-0.45\tau_B^{-1}/a_1 - 572.5D/a_1 - 3.42b_5/a_1\right]. \tag{54}$$

This is similar to an expression obtained by Saleh et al [16].

4. Discussion

In the frame of the Callaway integral as well as the generalized Callaway integral and assuming the additive nature of scattering relaxation rates, analytical expressions for the correction term are rederived for a lattice having dislocation for two different approximations. These expressions are very simple and easily computable in comparison with the complicated phenomenological expressions given by Callaway.

With the help of Eqs (16), (20), (42) and (53), it is obvious that the expression of ΔK is mainly governed by the three phonon normal scattering process. At the same time, with the help of Eqs (20) and (42), it is clear that for $\tau_B^{-1} \gg (\tau_{cd}^{-1} + \tau_{sd}^{-1})$, the correction term shows a $b_N \tau_B^2$ dependence. By examining Eqs (16) and (53) it can be seen that for $\tau_{sd}^{-1} \gg (\tau_B^{-1} + \tau_{cd}^{-1})$, $\Delta K \propto b_N / a_1^2$, which suggests that the correction term is mainly governed by the three phonon normal process and strain field dislocation scattering relaxation rate. These results obtained show similarity with the previous findings of earlier workers for the correction term [4-6, 11-13, 16].

References

1. J. Callaway, Phys. Rev., *113*, 1046, 1959.
2. V. V. Kosarev, P. V. Tamarin and S. S. Shalyt, Phys. Stat. Sol., (b)*44*, 525, 1971.
3. J. E. Parrot, Phys. Stat. Sol., (b)*48*, K159, 1971.
4. K. S. Dubey, Phys. Stat. Sol., (b)*63*, K35, 1974.
5. Y. P. Joshi, M. D. Tiwari and G. S. Verma, Physica, *47*, 231, 1970.
6. K. S. Dubey, Indian J. Pure and Appl. Phys., *11*, 265, 1973.
7. K. S. Dubey, J. de Physique, *37*, 267, 1976.
8. K. S. Dubey, Phys. Stat. Sol., (b)*81*, K83, 1977.
9. B. K. Agrawal and G. S. Verma, Phys. Rev., *128*, 603, 1962.
10. J. Callaway and M. C. V. Bayer, Phys. Rev., *120*, 1149, 1960.
11. K. S. Dubey, Solid State Commun., *23*, 963, 1977.
12. A. H. Awad, Acta Phys. Hung., *67*, 211, 1990.
13. A. H. Awad and J. S. Saif, Phys. Stat. Sol., (b)*151*, 495, 1989.
14. H. B. G. Casimir, Physica, *5*, 595, 1938.
15. P. G. Klemens, Solid State Physics, *7*, 1, 1958.
16. A. F. Saleh, R. H. Misho and K. S. Dubey, Acta Phys. Hung., *47*, 325, 1979.